A LEVEL MATHS 9709 · PAPER 1 PURE MATHEMATICS 1 · 1.4 · ~6 MIN READ

Circular measure: radians, arcs and sectors

What a radian really is, converting between degrees and radians, arc length, sector area and segment area, and the multi-step sector problems that combine them with triangles.

The radian | Degrees ↔ radians | Arc length s = rθ | Sector area ½r²θ | Segment area | Composite shapes | Exact answers

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **One radian** is the angle at the centre of a circle when the arc length equals the radius. A full turn is 2π radians, so **π rad = 180°**.
- **With θ in radians**, arc length s = rθ and sector area A = ½r²θ. These formulas are wrong in degrees.
- **Segment = sector − triangle** = ½r²(θ − sin θ).
- Most exam questions combine a sector with a triangle: use the sine rule, cosine rule or right-angled trigonometry for the straight edges, and the radian formulas for the curved ones.

BY THE END YOU CAN

- Explain what a radian is and convert between degrees and radians.
- Use s = rθ and A = ½r²θ to find lengths, areas, perimeters and angles.
- Find segment areas and the areas and perimeters of composite shapes.
- Give exact answers in terms of π and surds when asked.

1 The radian

Definition and conversions

One radian — the angle subtended at the centre of a circle by an arc equal in length to the radius.

- **Why 2π for a full turn?** The circumference is 2πr, so it contains 2π arcs of length r. A full turn is 2π radians.

$$\pi \text{ rad} = 180^\circ$$ LEARN IT

degrees → radians: × π/180; radians → degrees: × 180/π

Degrees	30°	45°	60°	90°	120°	135°	150°	180°	270°	360°
Radians	π/6	π/4	π/3	π/2	2π/3	3π/4	5π/6	π	3π/2	2π

- **Example:** 0.8 rad = 0.8 × 180/π = 45.8°. 1 rad ≈ 57.3°.

EXAM TIP

Check your calculator mode before every trig calculation in this topic. Radian questions done in degree mode give wrong answers with no method marks for the value.

2 Arcs and sectors

The formulas

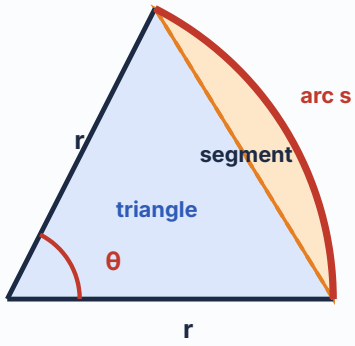
$s = r\theta$ **FORMULA LIST (MF19)**

arc length, θ in radians

$A = \frac{1}{2}r^2\theta$ **FORMULA LIST (MF19)**

sector area, θ in radians

- **Why they work:** a sector with angle θ is the fraction θ/(2π) of the whole circle. Arc = (θ/2π) × 2πr = rθ. Area = (θ/2π) × πr² = ½r²θ.
- **Perimeter of a sector** = r + r + rθ = 2r + rθ. Don't forget the two radii.



θ in radians

arc length s = rθ

sector area = ½r²θ

triangle area = ½r² sin θ

segment = ½r²(θ − sin θ)

The segment is the sector minus the triangle.

Sector, triangle and segment share the same two radii.

WORKED EXAMPLE

A sector has radius 8 cm and angle 1.2 radians. Find its arc length, perimeter and area.

1. arc s = 8 × 1.2 = **9.6 cm** B1
2. perimeter = 8 + 8 + 9.6 = **25.6 cm** B1
3. **area = ½ × 8² × 1.2 = 38.4 cm²** B1

WORKED EXAMPLE

A sector of a circle of radius 6 cm has area 30 cm². Find the angle of the sector.

1. ½ × 6² × θ = 30 M1
2. **θ = 60/36 = 5/3 rad (≈ 1.67 rad)** A1

3 Segments

Sector minus triangle

$$\text{segment area} = \tfrac{1}{2}r^2\theta - \tfrac{1}{2}r^2 \sin\theta = \tfrac{1}{2}r^2(\theta - \sin\theta)$$

θ in radians; the triangle uses ½ab sin C

WORKED EXAMPLE

A chord AB of a circle with centre O and radius 6 cm subtends angle π/3 at O. Find the exact area of the minor segment.

1. sector = ½ × 36 × π/3 = 6π **M1**
2. triangle = ½ × 36 × sin(π/3) = 18 × √3/2 = 9√3 **M1**
3. **segment = 6π – 9√3 cm² (≈ 3.26 cm²)** **A1**

- **Chord length:** for angle θ at the centre, chord = 2r sin(θ/2). Or use the cosine rule: chord² = r² + r² – 2r² cos θ.

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4 Composite shapes

Typical exam set-up

- **Draw and label** the diagram, marking every right angle (a tangent meets a radius at 90°).
- **Find straight lengths** with trigonometry: SOH CAH TOA in right-angled triangles, the sine or cosine rule otherwise.
- **Find curved lengths and areas** with $s = r\theta$ and $\frac{1}{2}r^2\theta$.
- **Add or subtract** regions. Write which ones: "shaded area = triangle OAC − sector OAB".

WORKED EXAMPLE

OAB is a sector of a circle with centre O, radius 10 cm and angle AOB = 0.9 rad. The tangent to the arc at A meets OB extended at C. Find the area and perimeter of the region bounded by AC, CB and the arc AB.

1. angle OAC = π/2 (tangent ⊥ radius), so AC = 10 tan 0.9 = 12.60 cm and OC = 10/cos 0.9 = 16.09 cm M1 A1
2. area = triangle OAC − sector OAB = ½ × 10 × 12.60 − ½ × 10^2 × 0.9 M1
3. = 63.01 − 45 = **18.0 cm²** A1
4. perimeter = AC + BC + arc AB = 12.60 + (16.09 − 10) + 10 × 0.9 M1
5. **= 27.7 cm** A1

COMMON MISTAKE

Stopping too early. In June 2024, many candidates found a perimeter or an area of one piece and stopped. Re-read the question: "the region bounded by..." usually needs two or three pieces.

COMMON MISTAKE

Rounding in the middle. Keep 12.6015... in your calculator; round only the final answer to 3 significant figures.

LINKING QUESTION

Radians are used for every calculus result with trig functions in Further differentiation, and for all trig graphs in Trigonometry.

Test yourself — cover the answers first

QUICK CHECK

1. Convert 225° to radians.
2. Convert 2.5 rad to degrees.
3. What is the arc length of a sector with radius 4 and angle π/4?
4. Write a formula for the area of a segment with angle θ at the centre.

EXAM-STYLE

5. A sector of radius r has perimeter 20 cm and area 16 cm². Find the possible values of r and θ.
6. A chord of a circle of radius 5 cm subtends an angle of 2π/3 at the centre. Find the area of the minor segment and the perimeter of the segment.

Show answers

1. $5\pi/4$
2. $143.2°$
3. π
4. $\frac{1}{2}r^2(\theta - \sin\theta)$
5. $2r + r\theta = 20$ and $\frac{1}{2}r^2\theta = 16 \Rightarrow r^2 - 10r + 16 = 0 \Rightarrow r = 2$ or 8
   $r = 2$ gives $\theta = 8$ rad, more than a full turn ($2\pi \approx 6.28$), so reject it
   $r = 8$ cm, $\theta = 0.5$ rad
6. area $= 12.5(2\pi/3 - \sin 2\pi/3) = 15.4$ cm^2; perimeter $= 10\pi/3 + 5\sqrt{3} = 19.1$ cm

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

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