A LEVEL MATHS 9709 · PAPER 6 PROBABILITY & STATISTICS 2 · 6.3 · ~3 MIN READ

Continuous random variables

Probability density functions, the area condition ∫f(x) dx = 1, probabilities as areas, the mean and variance by integration, and the median and other percentiles of a continuous random variable.

Probability density function | Total area = 1 | Probabilities as areas | Sketching pdfs | E(X) | Var(X) | Median | Percentiles

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

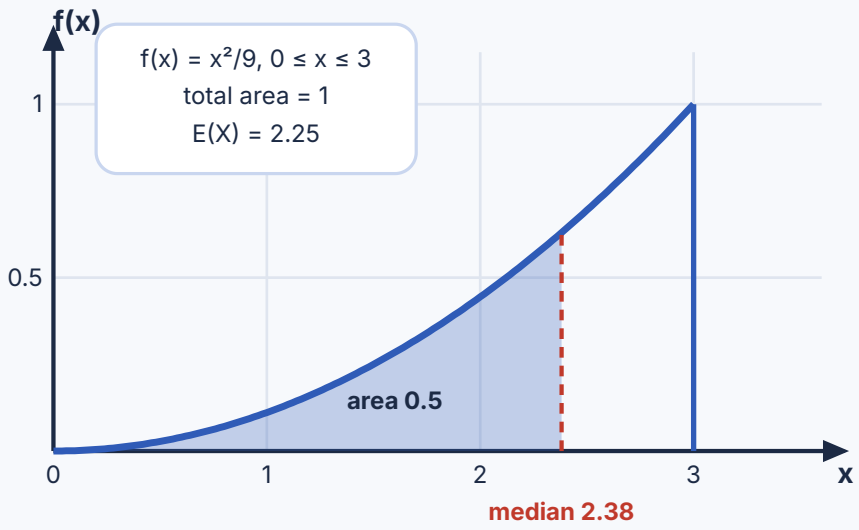
- **A pdf f(x) is never negative and the total area under it is 1.** Probabilities are areas: $P(a < X < b) = \int_a^b f(x)\,dx$.
- **For a continuous variable, P(X = a) = 0,** so < and ≤ give the same probability.
- **$E(X) = \int x f(x)\,dx$ and $Var(X) = \int x^2 f(x)\,dx - [E(X)]^2$,** over the whole range.
- **Median m:** solve ∫ from the lower limit to m of f(x) dx = 0.5.

BY THE END YOU CAN

- Use a pdf, including finding an unknown constant from the total area.
- Find probabilities by integration, and sketch simple pdfs.
- Calculate E(X) and Var(X).
- Find the median and other percentiles.

1 Probability density functions

The area rule



f(x) = x²/9 on [0, 3]: area under the curve is 1; the shaded area is a probability.

$$\int f(x)\,dx = 1 \text{ (over the whole range)} \; ; \; P(a < X < b) = \int_a^b f(x)\,dx$$

f(x) = 0 outside the stated range

WORKED EXAMPLE

f(x) = k(4 − x²) for 0 ≤ x ≤ 2, and 0 otherwise. Find k and E(X).

1. $k[4x - x^3/3]_0^2 = k(8 - 8/3) = 16k/3 = 1 \Rightarrow$ **k = 3/16** M1 A1
2. **$E(X) = (3/16)\int_0^2 (4x - x^3)\,dx = (3/16)(8 - 4) = 0.75$** M1 A1

COMMON MISTAKE

Ignoring the domain. Integrate only over the range where f(x) is non-zero.

2 Mean, variance and median

By integration

$$E(X) = \int x\,f(x)\,dx \; ; \; Var(X) = \int x^2 f(x)\,dx - [E(X)]^2$$ FORMULA LIST (MF19)

integrate over the whole range

WORKED EXAMPLE

f(x) = x²/9 for 0 ≤ x ≤ 3. Find (a) E(X), (b) Var(X), (c) the median, (d) P(X < 1).

1. (a) $\int_0^3 x^3/9\,dx = 81/36 =$ **2.25** M1 A1
2. (b) $\int_0^3 x^4/9\,dx = 243/45 = 5.4$; Var = $5.4 - 2.25^2 =$ **0.3375** M1 A1
3. (c) $m^3/27 = 0.5 \Rightarrow m = 3 \times 0.5^{1/3} =$ **2.38** M1 A1
4. **(d) 1/27 = 0.0370** B1

- **The median here (2.38) exceeds the mean (2.25):** the pdf is increasing, with a long tail to the left (negative skew).
- **Percentiles:** for the upper quartile solve the integral = 0.75.
- **Mode:** the x where f(x) is greatest (look at the sketch, or set f′(x) = 0).

COMMON MISTAKE

Forgetting to subtract the mean squared, or using f(x) instead of xf(x) for the mean.

LINKING QUESTION

The integration is from Integration and Further integration; the normal distribution in The normal distribution is the most important continuous distribution.

Test yourself — cover the answers first

QUICK CHECK

1. What must the area under a pdf equal?
2. Find P(X = 1.5) for a continuous X.
3. f(x) = ½ for 0 ≤ x ≤ 2. Find E(X).

4. Write the equation for the median m of f on [0, 4].

5. Can a pdf take the value 1.4 at some point?

EXAM-STYLE

6. $f(x) = k/x^2$ for $1 \le x \le 4$. Find k and the median.

7. $f(x) = 2x$ for $0 \le x \le 1$. Find Var(X).

8. For $f(x) = x^2/9$ on [0, 3], find the upper quartile.

Show answers

1. 1
2. 0
3. 1
4. $\int_0^m f(x)\,dx = 0.5$
5. yes, only areas must be at most 1
6. $k(1 - \frac{1}{4}) = 1 \Rightarrow k = 4/3$; $(4/3)(1 - 1/m) = 0.5 \Rightarrow m = 1.6$
7. $E(X) = 2/3$, $E(X^2) = 1/2$, $\text{Var} = 1/2 - 4/9 = 1/18$
8. $q^3/27 = 0.75 \Rightarrow q = 3 \times 0.75^{1/3} = 2.73$

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Share with a friend | Ask Riyaz Sir

Want regular help? Send this to your parent →