A LEVEL MATHS 9709 · PAPER 1 PURE MATHEMATICS 1 · 1.3 · ~8 MIN READ

Coordinate geometry: lines and circles

Equations of straight lines, parallel and perpendicular gradients, perpendicular bisectors, the equation of a circle in both forms, tangents, chords and where lines meet circles.

Gradient & line equations | Parallel & perpendicular | Midpoint & distance | Perpendicular bisector | Circle equation | Centre & radius | Tangents | Line meets circle

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **A straight line** needs a point and a gradient. Use $y - y_1 = m(x - x_1)$: it is quicker and safer than $y = mx + c$.
- **Perpendicular gradients multiply to −1:** if one gradient is m, the other is $-1/m$.
- **A circle with centre (a, b) and radius r** is $(x - a)^2 + (y - b)^2 = r^2$. To find the centre and radius from the expanded form, complete the square in x and in y.
- **Three circle facts do most of the work:** a tangent is perpendicular to the radius; the angle in a semicircle is 90°; the perpendicular bisector of any chord passes through the centre.

BY THE END YOU CAN

- Find the equation of a line from two points, or a point and a gradient, in any required form.
- Use parallel and perpendicular gradients, midpoints and distances.
- Find the centre and radius of a circle and write its equation.
- Find tangents to circles and decide whether a line cuts, touches or misses a circle.

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1 Straight lines

The toolkit

$$m = (y_2 - y_1)/(x_2 - x_1)$$

gradient through (x_1, y_1) and (x_2, y_2)

$$y - y_1 = m(x - x_1)$$

line through (x_1, y_1) with gradient m

- **Other forms:** y = mx + c (gradient m, y-intercept c); ax + by + c = 0 (rearrange to read off m = −a/b).
- **Midpoint** of (x_1, y_1) and (x_2, y_2): $((x_1 + x_2)/2, (y_1 + y_2)/2)$. **Distance:** $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$.
- **Parallel lines** have equal gradients. **Perpendicular lines:** $m_1m_2 = -1$ LEARN IT.
- **Intersection of two lines:** solve the equations simultaneously.

WORKED EXAMPLE

Find the equation of the perpendicular bisector of A(−1, 3) and B(5, 7), in the form ax + by = c.

1. midpoint M = (2, 5) B1
2. gradient AB = (7 − 3)/(5 − (−1)) = 4/6 = ⅔, so the perpendicular gradient is −3/2 M1
3. y − 5 = −3/2(x − 2) M1
4. 2y − 10 = −3x + 6 ⇒ **3x + 2y = 16** A1
5. **check: M(2, 5) gives 6 + 10 = 16 ✓**

COMMON MISTAKE

Using the curve's gradient for the perpendicular. In 2024 some candidates took the perpendicular gradient as ⅓ from a gradient of 3 (the reciprocal, without the minus sign). The perpendicular gradient is −1/m.

EXAM TIP

The **perpendicular bisector** of AB is the set of points equidistant from A and B. It passes through the midpoint with gradient −1/m_AB. If A and B lie on a circle, it passes through the centre.

2 Circles

Two forms of the equation

$$(x - a)^2 + (y - b)^2 = r^2$$ **LEARN IT**

centre (a, b), radius r (it is Pythagoras: the distance from (x, y) to the centre is r)

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

centre $(-g, -f)$, radius $\sqrt{(g^2 + f^2 - c)}$

- **From expanded to centre–radius form:** complete the square in x and in y separately. This is safer than memorising the second form.

WORKED EXAMPLE

Find the centre and radius of the circle $x^2 + y^2 - 6x + 4y - 12 = 0$.

1. $(x - 3)^2 - 9 + (y + 2)^2 - 4 - 12 = 0$ M1
2. $(x - 3)^2 + (y + 2)^2 = 25$
3. **centre (3, −2), radius 5** A1 A1

WORKED EXAMPLE

A and B have coordinates (1, 2) and (7, 10). Find the equation of the circle with diameter AB.

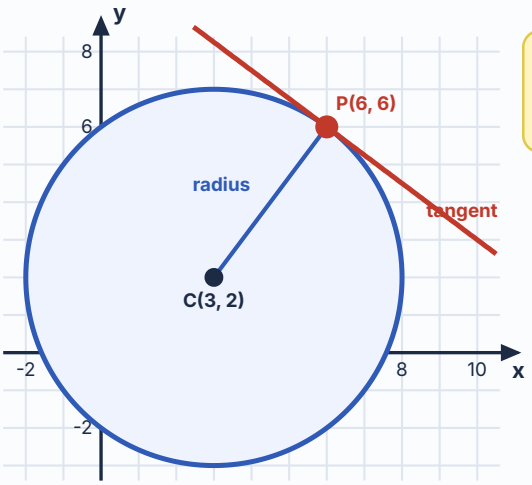
1. centre = midpoint of AB = (4, 6) B1
2. radius = ½AB = ½√(6² + 8²) = ½ × 10 = 5 M1
3. **$(x - 4)^2 + (y - 6)^2 = 25$** A1

COMMON MISTAKE

Sign errors with the centre. $(x + 2)^2$ means the centre has x-coordinate −2, not 2.

Circle properties

Property	How it is used
A tangent is perpendicular to the radius at the point of contact	tangent gradient = −1/(gradient of radius)
The angle in a semicircle is 90°	if AB is a diameter and P is on the circle, AP ⊥ BP
The perpendicular from the centre to a chord bisects the chord	the centre lies on the perpendicular bisector of every chord
Symmetry about any diameter	reflect known points in a line through the centre



gradient CP = 4/3
tangent gradient = −3/4
perpendicular: $m_1m_2 = -1$

$(x - 3)^2 + (y - 2)^2 = 25$. The tangent at P is perpendicular to the radius CP.

At P(6, 6), the radius has gradient 4/3, so the tangent has gradient −3/4.

WORKED EXAMPLE

The point P(6, 2) lies on the circle $(x - 3)^2 + (y + 2)^2 = 25$. Find the equation of the tangent at P.

1. gradient of radius CP = $(2 - (-2))/(6 - 3) = 4/3$ M1
2. tangent gradient = $-3/4$ M1
3. $y - 2 = -\frac{3}{4}(x - 6)$ M1
4. **$4y - 8 = -3x + 18 \Rightarrow 3x + 4y = 26$** A1

- **Circle through three points A, B, C:** the centre is where the perpendicular bisectors of AB and BC meet. Then the radius is the distance from the centre to any of the points.

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3 Where a line meets a circle

Substitute and use the discriminant

- **Substitute the line into the circle** to get a quadratic in x (or y). Solve it for the intersection points.
- **The discriminant tells you the geometry:** > 0 the line is a chord (two points); = 0 it is a tangent; < 0 it misses.

WORKED EXAMPLE

Find where the line $y = 2x - 3$ meets the circle $(x - 3)^2 + (y + 2)^2 = 25$, and the length of the chord.

1. $(x - 3)^2 + (2x - 3 + 2)^2 = 25 \Rightarrow (x - 3)^2 + (2x - 1)^2 = 25$ M1
2. $5x^2 - 10x + 10 = 25 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow (x - 3)(x + 1) = 0$ A1
3. points **(3, 3) and (−1, −5)** A1
4. **chord length $= \sqrt{(4^2 + 8^2)} = \sqrt{80} = 4\sqrt{5}$** B1

WORKED EXAMPLE

Find the values of c for which the line $y = 2x + c$ is a tangent to the circle $x^2 + y^2 = 5$.

1. $x^2 + (2x + c)^2 = 5 \Rightarrow 5x^2 + 4cx + (c^2 - 5) = 0$ M1
2. tangent ⇒ discriminant = 0: $16c^2 - 20(c^2 - 5) = 0$ M1
3. $100 - 4c^2 = 0 \Rightarrow$ **$c = \pm 5$** A1
4. **check: the distance from the centre (0, 0) to $2x - y + c = 0$ is $|c|/\sqrt{5} = \sqrt{5} = r$ ✓**

- **Relating graphs and equations:** "Find the set of values of k for which $y = x + k$ meets the curve" means "find k so that the discriminant is ≥ 0". See Quadratics.

COMMON MISTAKE

Substituting into the circle and forgetting to square the whole bracket: $(2x - 1)^2 = 4x^2 - 4x + 1$, not $4x^2 + 1$.

LINKING QUESTION

The discriminant method is from Quadratics; perpendicular gradients give normals to curves in Differentiation; and the same circle equations in complex form are loci in Complex numbers.

Test yourself — cover the answers first

QUICK CHECK

1. What is the gradient of a line perpendicular to $2x + 5y = 7$?
2. Find the centre and radius of $(x + 1)^2 + (y - 4)^2 = 49$.
3. Write down the condition for AP ⊥ BP when AB is a diameter of a circle through P.
4. How do you find the centre of a circle through three given points?

EXAM-STYLE

5. The points A(2, 1) and B(8, 5) lie on a circle whose centre lies on the line $y = x - 2$. Find the equation of the circle.
6. The line $x + y = k$ is a tangent to the circle $x^2 + y^2 - 4x - 6y + 9 = 0$. Find the possible values of k.
7. Find the equation of the tangent to $x^2 + y^2 - 2x + 4y - 20 = 0$ at the point (4, 2).

Show answers

1. 5/2
2. (−1, 4), 7
3. angle in a semicircle is 90°
4. intersect the perpendicular bisectors of two chords
5. perpendicular bisector of AB: $3x + 2y = 21$; with $y = x - 2$: centre (5, 3); $r^2 = 13$; $(x - 5)^2 + (y - 3)^2 = 13$
6. centre (2, 3), $r = 2$; $|5 - k|/\sqrt{2} = 2 \Rightarrow k = 5 \pm 2\sqrt{2}$
7. centre (1, −2); radius gradient 4/3; $3x + 4y = 20$

Stuck on a question?

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