A LEVEL MATHS 9709 · PAPER 5 PROBABILITY & STATISTICS 1 · 5.4 · ~4 MIN READ

Discrete random variables

Probability distribution tables, expectation and variance of a discrete random variable, and the binomial and geometric distributions: when to use them, their probabilities, means and variances.

Probability distributions | Σp = 1 | E(X) | Var(X) | Binomial B(n, p) | Conditions for binomial | Geometric Geo(p) | Mean & variance formulae

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **A probability distribution lists every value with its probability;** the probabilities add up to 1.
- **E(X) = Σx p** and **Var(X) = Σx² p – [E(X)]².**
- **Binomial B(n, p):** a fixed number n of independent trials, each a success with the same probability p. X counts the successes.
- **Geometric Geo(p):** independent trials with constant p, until the first success. X is the trial on which it happens.

BY THE END YOU CAN

- Draw up a probability distribution table and use Σp = 1.
- Calculate E(X) and Var(X) for a discrete random variable.
- Recognise and use the binomial distribution, including its mean np and variance np(1 – p).
- Recognise and use the geometric distribution, including P(X > r) and its mean 1/p.

1 Probability distributions

Expectation and variance

$$E(X) = \Sigma x\, P(X = x)\ ;\ Var(X) = \Sigma x^2\, P(X = x) - [E(X)]^2$$ FORMULA LIST (MF19)

the standard deviation is √Var(X)

WORKED EXAMPLE

P(X = x) = kx for x = 1, 2, 3, 4. Find k, E(X) and Var(X).

1. k(1 + 2 + 3 + 4) = 1 ⇒ **k = 1/10** B1
2. E(X) = (1 + 4 + 9 + 16)/10 = **3** M1 A1
3. **E(X²) = (1 + 8 + 27 + 64)/10 = 10 ⇒ Var(X) = 10 − 9 = 1** M1 A1

COMMON MISTAKE

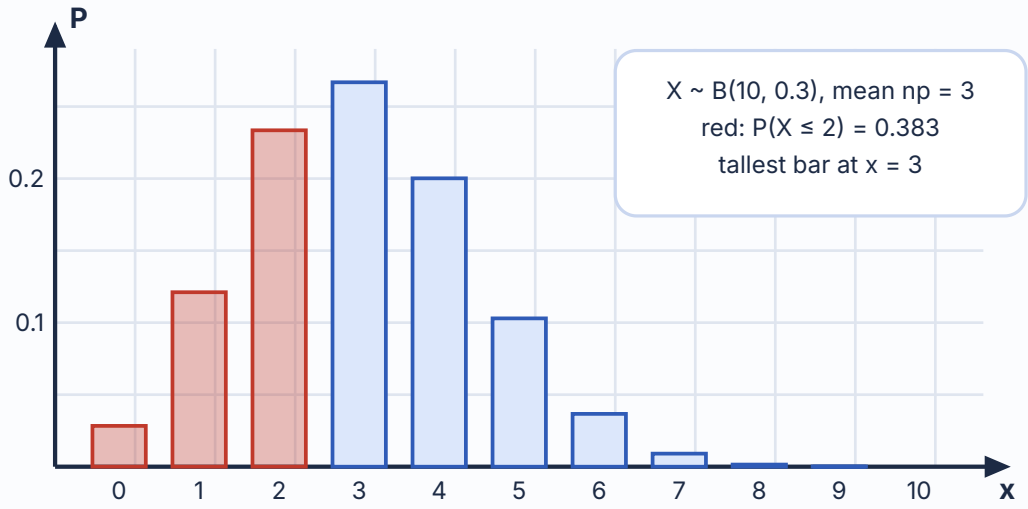
Var(X) = Σx²p with the mean not subtracted, or squaring the probabilities instead of the x-values.

2 The binomial distribution

X ~ B(n, p)

$$P(X = r) = {}^{n}C_{r}\, p^{r}\, (1-p)^{n-r} \; ; \; E(X) = np \; ; \; Var(X) = np(1-p)$$ FORMULA LIST (MF19)

$r = 0, 1, \ldots, n$



B(10, 0.3): the peak is near the mean np = 3.

- **Conditions in context:** "each item is faulty independently with the same probability". Questions often ask you to state one.

WORKED EXAMPLE

X ~ B(10, 0.3). Find (a) P(X = 3), (b) P(X ≤ 2), (c) P(X ≥ 2).

1. (a) ${}^{10}C_{3}\,(0.3)^{3}(0.7)^{7}$ = **0.267** M1 A1
2. (b) P(0) + P(1) + P(2) = 0.0282 + 0.1211 + 0.2335 = **0.383** M1 A1
3. **(c) 1 − P(0) − P(1) = 0.851** M1 A1

COMMON MISTAKE

"At least 2" read as 1 − P(X ≤ 2). "At least 2" means X ≥ 2, so subtract P(X ≤ 1). Write the inequality before calculating.

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3 The geometric distribution

X ~ Geo(p)

$$P(X = r) = p(1-p)^{r-1}\ ;\ P(X > r) = (1-p)^r\ ;\ E(X) = 1/p$$

$r = 1, 2, 3, \ldots$ (no upper limit)

- **Why $P(X > r) = (1-p)^r$:** the first r trials all fail.

WORKED EXAMPLE

A fair die is thrown until a six appears. Find (a) P(X = 4), (b) P(X > 3), (c) E(X).

1. (a) $(5/6)^3(1/6)$ = **0.0965** M1 A1
2. (b) $(5/6)^3$ = **0.579** B1
3. **(c) 6** B1

WORKED EXAMPLE

Find the least number of throws needed for the probability of at least one six to exceed 0.9.

1. $1 - (5/6)^n > 0.9 \Rightarrow (5/6)^n < 0.1$ M1
2. $n > \ln 0.1/\ln(5/6) = 12.6$ M1
3. **n = 13** A1

COMMON MISTAKE

Using binomial for "until the first success". The number of trials isn't fixed, so it's geometric.

LINKING QUESTION

Probability rules are in Probability; ${}^nC_r$ is in Permutations and combinations; large binomials are approximated in The normal distribution; and the Poisson distribution is in The Poisson distribution.

Test yourself — cover the answers first

QUICK CHECK

1. X ~ B(20, 0.25). Find E(X).
2. X ~ B(20, 0.25). Find Var(X).
3. X ~ Geo(0.2). Find E(X).
4. X ~ Geo(0.2). Find P(X = 2).
5. State two conditions for a binomial model.

EXAM-STYLE

6. X ~ B(8, 0.4). Find $P(2 < X \leq 5)$.
7. The probability that a seed germinates is 0.85. 12 seeds are planted. Find the probability that at least 11 germinate.
8. X ~ Geo(p) and P(X = 1) = 0.3. Find $P(X \geq 3)$.

Show answers

1. 5
2. 3.75
3. 5
4. 0.16
5. fixed number of independent trials; constant probability of success
6. P(3) + P(4) + P(5) = 0.2787 + 0.2322 + 0.1239 = 0.635
7. $12(0.85)^{11}(0.15) + 0.85^{12} = 0.3012 + 0.1422 = 0.443$
8. $(0.7)^2 = 0.49$

Stuck on a question?

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