A LEVEL MATHS 9709 · PAPER 4 MECHANICS · 4.1 · ~6 MIN READ

# Forces and equilibrium

Identifying the forces on a particle, resolving them into components, using the conditions for equilibrium, finding resultants, and modelling friction with F ≤ μR, including limiting equilibrium on rough horizontal and inclined surfaces.

Types of force | Force diagrams | Resolving | Resultant force | Equilibrium of a particle | Friction & μ | Limiting equilibrium | Inclined planes

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

## IN 30 SECONDS

- **Draw the force diagram first:** weight mg down, normal reaction R perpendicular to the surface, friction F along the surface opposing (possible) motion, and any tensions or applied forces.
- **Resolve in two perpendicular directions.** On a slope, use along and perpendicular to the slope.
- **Equilibrium:** the components in every direction sum to zero. That gives two equations for two unknowns.
- **Friction: F ≤ μR.** F = μR only when the particle is moving or about to move (limiting equilibrium).

## BY THE END YOU CAN

- Identify the forces acting on a particle and draw a clear diagram.
- Find the resultant of several forces by resolving.
- Use the conditions for equilibrium of a particle in two dimensions.
- Model friction with F ≤ μR and use limiting friction, including on inclined planes.

## 1 Forces and resolving

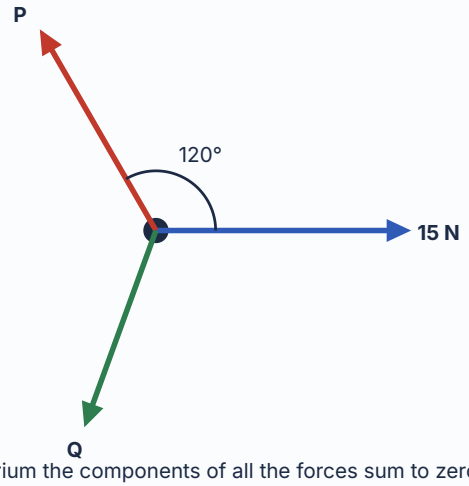
### The language of mechanics

| Term | Meaning in a model |
|---|---|
| Particle | an object whose size is ignored; all forces act at one point |
| Light (string, rod) | its mass is ignored, so tension is the same throughout |
| Inextensible | its length doesn't change, so connected particles share an acceleration |
| Smooth | no friction |
| Rough | friction acts, up to a limit μR |

- **Use g = 10 m s$^{-2}$ in this paper,** unless the question says otherwise. (9702 Physics uses 9.81.)
- **Resolving:** a force P at angle θ to a direction has component P cos θ along it and P sin θ perpendicular to it.

$$\text{Resultant: } X = \Sigma P\cos\theta,\ Y = \Sigma P\sin\theta\ ;\ |R| = \sqrt{(X^2 + Y^2)},\ \tan\varphi = Y/X$$

resolve every force, add, then recombine




resolve in two directions: each sum = 0 ⇒ 2 equations for 2 unknowns

In equilibrium the components of all the forces sum to zero in every direction.

Three forces in equilibrium: the components in each direction cancel.

WORKED EXAMPLE

**Forces of 15 N, P N and Q N act at a point at angles 0°, 120° and 250° to the positive x-axis, and are in equilibrium. Find P and Q.**

1. x: 15 + P cos 120° + Q cos 250° = 0 M1
2. y: P sin 120° + Q sin 250° = 0 M1
3. **solving: P = 18.4, Q = 17.0** A1 A1

COMMON MISTAKE

**Mixing sin and cos.** The component next to the angle uses cos. Draw the angle on your diagram before resolving.

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## 2 Friction

### $F \leq \mu R$

**Coefficient of friction μ** — the ratio of limiting friction to normal reaction. A smooth surface has μ = 0.

- **Limiting equilibrium:** "on the point of moving" or "about to slip" ⇒ F = μR.
- **Not yet limiting:** F is just whatever is needed for equilibrium, and F < μR.
- **Normal reaction is not always mg.** A pull at an angle above the horizontal reduces R; a push down increases it.

WORKED EXAMPLE

**A box of mass 5 kg rests on rough horizontal ground. A force of 30 N at 20° above the horizontal makes it just about to move. Find μ.**

1. vertical: R + 30 sin 20° = 50 ⇒ R = 39.74 N M1 A1
2. horizontal: F = 30 cos 20° = 28.19 N B1
3. **limiting: μ = F/R = 0.709** M1 A1

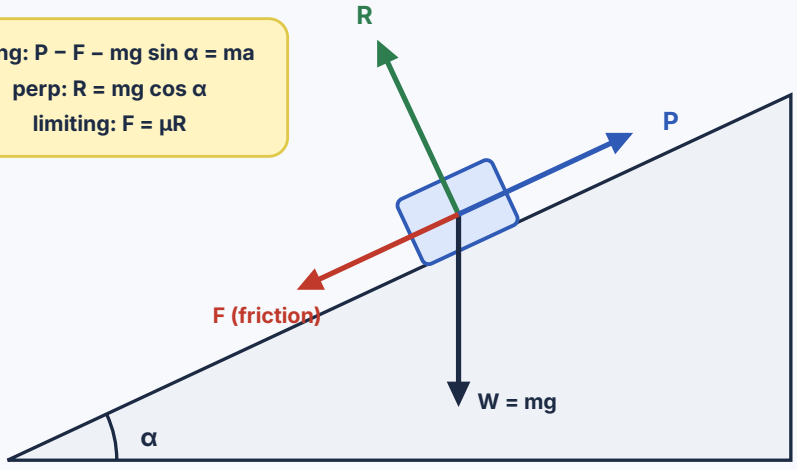
COMMON MISTAKE

**Writing R = mg** when another force has a vertical component. The 2024 Paper 4 report singled this out.

## 3 Inclined planes

### Along and perpendicular

along: P − F − mg sin α = ma
perp: R = mg cos α
limiting: F = μR




Resolve parallel and perpendicular to the slope. Friction opposes the (possible) motion.

Resolve along and perpendicular to the slope; friction acts along the slope.

Along the slope: mg sin α ; perpendicular: R = mg cos α (if no other perpendicular forces)

α is the angle of the slope

**WORKED EXAMPLE**

**A particle of mass 2 kg is on a rough plane inclined at 25°, with μ = 0.3. (a) Show it would slide down if released. (b) Find the least force P, acting up the slope, that keeps it at rest. (c) Find the force P up the slope that makes it about to move up.**

1. (a) down-slope component 20 sin 25° = 8.45 N; maximum friction 0.3 × 20 cos 25° = 5.44 N; 8.45 > 5.44 so it slides M1 A1
2. (b) friction acts up the slope: P + 5.44 = 8.45 ⇒ **P = 3.01 N** M1 A1
3. **(c) friction acts down the slope: P = 8.45 + 5.44 = 13.9 N** M1 A1

- **Which way does friction act?** Opposite to the motion that would happen. In (b) the particle tends to slide down; in (c) it tends to move up. The two answers give the range of P for equilibrium.

**COMMON MISTAKE**

**Friction pointing the wrong way,** usually because the direction of possible motion wasn't thought about first.

**LINKING QUESTION**

The forces here produce accelerations in Newton's laws of motion; work against friction is in Energy, work and power; and 3D vectors are in Vectors.

## Test yourself — cover the answers first

**QUICK CHECK**

1. What does "light" tell you about a string?
2. A 4 kg mass is on a smooth slope at 30°. Find the component of weight down the slope.
3. When is F = μR?
4. What is the normal reaction on a 3 kg block on a horizontal floor with no other vertical forces?
5. Find the resultant of 6 N east and 8 N north.

**EXAM-STYLE**

6. A 4 kg particle rests on a rough plane inclined at 30° and is on the point of slipping down. Find μ.
7. A sledge of mass 12 kg is pulled at constant speed along rough horizontal ground by a rope at 30° above the horizontal with tension 50 N. Find μ.
8. A particle hangs in equilibrium from two strings at 30° and 60° to the horizontal. Its weight is 20 N. Find the tensions.

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Show answers

1. its mass is ignored, so the tension is the same throughout
2. 40 sin 30° = 20 N
3. when moving or on the point of moving
4. 30 N
5. 10 N at 53.1° north of east
6. 40 sin 30° = μ × 40 cos 30° ⇒ μ = tan 30° = 0.577
7. R = 120 − 25 = 95; F = 50 cos 30° = 43.3; μ = 0.456
8. $T_1 \cos 30° = T_2 \cos 60°$; $T_1 \sin 30° + T_2 \sin 60° = 20$ ⇒ $T_1 = 10$ N, $T_2 = 17.3$ N


**Stuck on a question?**

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