A LEVEL MATHS 9709 · PAPER 6 PROBABILITY & STATISTICS 2 · 6.5 · ~5 MIN READ

Hypothesis tests

Null and alternative hypotheses, significance levels, one- and two-tailed tests, critical regions, tests for a binomial proportion, a Poisson mean and a normal mean, and Type I and Type II errors with their probabilities.

H_0 and H_1 | Significance level | One- & two-tailed | Critical regions | Binomial test | Poisson test | Normal mean test | Type I error

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **H_0 states the accepted value** ($p = 0.3$, $\lambda = 6$, $\mu = 50$); **H_1 states what you suspect** ($<$, $>$ or $\neq$).
- **Reject H_0 if the result is in the critical region,** i.e. the probability of a result this extreme (assuming H_0) is less than the significance level. For two-tailed tests, compare with half the level in each tail.
- **Discrete tests** (binomial, Poisson) use cumulative probabilities; **normal tests** use a z-value.
- **Type I error:** rejecting H_0 when it is true; its probability is the actual significance level. **Type II error:** accepting H_0 when it is false.

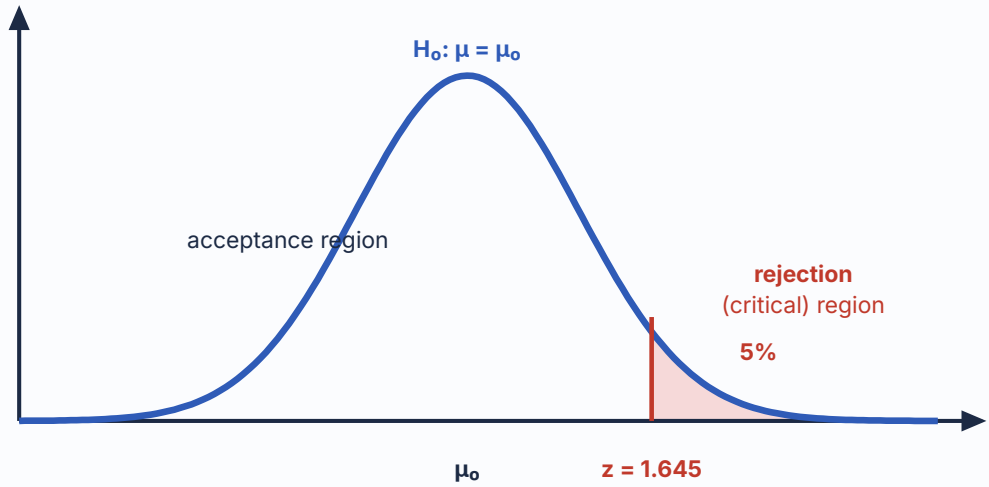
BY THE END YOU CAN

- State hypotheses in terms of a population parameter.
- Carry out tests for a binomial proportion, a Poisson mean and a normal mean, one- or two-tailed.
- Find critical regions and actual significance levels for discrete tests.
- Understand Type I and Type II errors and calculate their probabilities.
- Write conclusions in context, without overstating them.

1 The language of tests

Hypotheses and conclusions

- **Hypotheses use parameters:** "H_0: $p = 0.3$, H_1: $p < 0.3$", not "H_0: $X = 2$".
- **Conclusions are cautious:** "there is evidence at the 5% level that the proportion has decreased", or "insufficient evidence that ...". Never "this proves".
- **Significance level:** the probability of rejecting H_0 when it's true. Smaller levels need stronger evidence.



One-tailed test at 5%: reject H_o if the test statistic lies in the shaded tail.

One-tailed normal test at 5%: reject H_o if z lies in the shaded tail.

COMMON MISTAKE

A conclusion with no context, or a definite claim. The 2024 Paper 6 report: a contradictory or definite conclusion loses the final mark.

2 Discrete tests

Binomial

WORKED EXAMPLE

A shop claims 30% of customers pay by card. In a sample of 20, only 2 do. Test at the 5% level whether the proportion is less than 30%.

1. H_o: $p = 0.3$; H_1: $p < 0.3$ B1
2. under H_o, $X \sim B(20, 0.3)$: $P(X \leq 2) = 0.0355$ M1 A1
3. $0.0355 < 0.05$, so reject H_o M1
4. **there is evidence at the 5% level that the proportion paying by card is less than 30%** A1

WORKED EXAMPLE

For this test, find the critical region and the probability of a Type I error. If in fact $p = 0.1$, find the probability of a Type II error.

1. $P(X \leq 2) = 0.0355 < 0.05$ but $P(X \leq 3) = 0.1071 > 0.05$ M1
2. critical region $\mathbf{X \leq 2}$; P(Type I) = **0.0355** A1 A1
3. **Type II: accept H_o although $p = 0.1$: $P(X \geq 3 \mid p = 0.1) = 1 - 0.6769 = 0.323$** M1 A1

Poisson

WORKED EXAMPLE

A road had an average of 6 accidents a week. After a speed limit was introduced, there were 2 accidents in a week. Test at the 5% level whether the rate has decreased.

1. H_0: $\lambda = 6$; H_1: $\lambda < 6$ **B1**
2. $P(X \leq 2) = e^{-6}(1 + 6 + 18) = 0.0620$ **M1** **A1**
3. $0.0620 > 0.05$, so do not reject H_0 **M1**
4. **insufficient evidence at the 5% level that the accident rate has decreased** **A1**

COMMON MISTAKE

Comparing P(X = 2) with the significance level. You need the tail probability, $P(X \leq 2)$.

3 Normal mean tests

z-tests

$$z = (\bar{x} - \mu_0)/(\sigma/\sqrt{n})$$

σ known, or n large with s in place of σ

WORKED EXAMPLE

Lengths are known to be normal with σ = 6. A sample of 36 has mean 48.1. Test at the 5% level whether the mean is less than 50.

1. H_0: $\mu = 50$; H_1: $\mu < 50$ **B1**
2. $z = (48.1 - 50)/(6/6) = -1.9$ **M1** **A1**
3. $-1.9 < -1.645$, so reject H_0 **M1**
4. **evidence at the 5% level that the mean length is less than 50** **A1**

- **Two-tailed at 5%:** critical values ±1.96. With $\bar{x} = 51.5$ instead, $z = 1.5$, between −1.96 and 1.96, so there's insufficient evidence that $\mu \neq 50$.

COMMON MISTAKE

Using σ instead of σ/√n, or the wrong critical value for a two-tailed test.

LINKING QUESTION

The distributions tested here are in Discrete random variables, The Poisson distribution and Sampling and estimation.

Test yourself — cover the answers first

QUICK CHECK

1. State the meaning of a Type I error.
2. What critical z-values are used in a two-tailed 5% test?
3. Write hypotheses for "has the mean increased from 20?".

4. A test statistic falls in the critical region. What do you do?

5. Why is the actual significance level of a binomial test often below 5%?

EXAM-STYLE

6. $X \sim B(15, 0.5)$ under H_0: $p = 0.5$ vs H_1: $p > 0.5$. Find the critical region at the 5% level.

7. Under H_0: $\lambda = 6$, with critical region $X \leq 1$, find P(Type I error).

8. $\sigma = 4$, $n = 25$, test H_0: $\mu = 30$ vs H_1: $\mu > 30$ at 1%. Find the smallest $\bar{x}$ that leads to rejection.

Show answers

1. rejecting H_0 when it is true
2. ±1.96
3. H_0: $\mu = 20$, H_1: $\mu > 20$
4. reject H_0
5. X is discrete, so the tail probability jumps
6. $P(X \geq 12) = 0.0176$, $P(X \geq 11) = 0.0592 \Rightarrow X \geq 12$
7. $e^{-6}(1 + 6) = 0.0174$
8. $30 + 2.326 \times 4/5 = 31.86$

Stuck on a question?

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Ask Riyaz Sir

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