A LEVEL MATHS 9709 · PAPER 4 MECHANICS · 4.2 · ~5 MIN READ

Kinematics of motion in a straight line

Distance, displacement, speed, velocity and acceleration, displacement-time and velocity-time graphs, the constant-acceleration (suvat) equations including vertical motion under gravity, and calculus for motion with variable acceleration.

Displacement vs distance | Velocity vs speed | s–t and v–t graphs | Area & gradient | suvat equations | Vertical motion | Variable acceleration | Differentiating & integrating

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

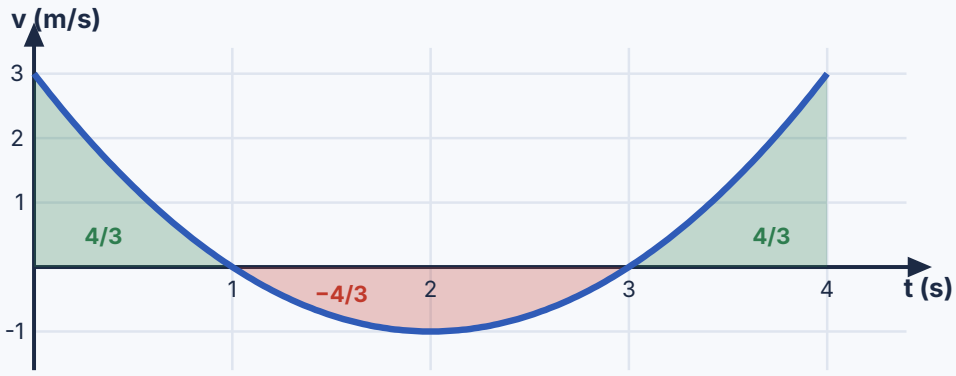
- **Vectors vs scalars:** displacement and velocity have direction (and can be negative); distance and speed don't.
- **v–t graph:** the gradient is the acceleration; the area under it is the displacement.
- **suvat works only when acceleration is constant.** List s, u, v, a, t, mark the three you know, and pick the equation without the one you don't want.
- **Variable acceleration:** differentiate s → v → a; integrate a → v → s, using initial conditions for the constants.

BY THE END YOU CAN

- Distinguish distance and displacement, speed and velocity.
- Sketch and interpret displacement-time and velocity-time graphs.
- Use the constant-acceleration formulae, including vertical motion under gravity (g = 10 m s^{-2}).
- Use differentiation and integration for motion with acceleration depending on time.

1 Graphs

Gradient and area



$v = t^2 - 4t + 3$: displacement = $\int v\,dt$ = 4/3 m, distance = $\int |v|\,dt$ = 4 m
Below the axis the particle is moving backwards.

On a v–t graph, gradient = acceleration and area = displacement.

Graph	Gradient	Area under it
displacement–time	velocity	(not used)
velocity–time	acceleration	displacement

- **Area below the t-axis** is negative displacement. For total distance, add the sizes of all the areas.

WORKED EXAMPLE

A train accelerates uniformly from rest to 12 m s^{-1} in 8 s, travels at 12 m s^{-1} for 20 s, then decelerates uniformly to rest in 6 s. Find the total distance.

1. area = ½ × 8 × 12 + 20 × 12 + ½ × 6 × 12 M1
2. **= 48 + 240 + 36 = 324 m** A1

COMMON MISTAKE

Reading a value off the graph as an area, or counting a trapezium as a rectangle. Sketch the shapes and label each one.

2 Constant acceleration

The suvat equations

$v = u + at$; $s = \frac{1}{2}(u + v)t$; $s = ut + \frac{1}{2}at^2$; $v^2 = u^2 + 2as$; $s = vt - \frac{1}{2}at^2$ FORMULA LIST (MF19)

a must be constant

WORKED EXAMPLE

A car moving at 4 m s^{-1} accelerates at 2.5 m s^{-2} for 6 s. Find its final speed and the distance covered.

1. v = 4 + 2.5 × 6 = **19 m s^{-1}** B1
2. **s = 4 × 6 + ½ × 2.5 × 36 = 69 m** M1 A1

Vertical motion

- **Choose a positive direction and stick to it.** With up positive, a = −10 and a point below the start has negative s.

WORKED EXAMPLE

A ball is thrown vertically upwards at 15 m s^{-1} from a point 20 m above the ground. Find (a) its greatest height above the ground and (b) the time it takes to hit the ground.

1. (a) at the top v = 0: 0 = 225 − 20s ⇒ s = 11.25 m, so **31.25 m** above ground M1 A1
2. (b) up positive: −20 = 15t − 5t² ⇒ t² − 3t − 4 = 0 M1
3. **(t − 4)(t + 1) = 0 ⇒ t = 4 s** A1

COMMON MISTAKE

Sign errors with a = −g. Forgetting that s is negative when the ball ends below its start. Mixing up directions is common, says the 2024 Paper 4 report.

3 Variable acceleration

Using calculus

$$v = \frac{ds}{dt} ; a = \frac{dv}{dt} = \frac{d^2s}{dt^2} ; s = \int v\,dt ; v = \int a\,dt$$

use these whenever a depends on t

- **At rest:** v = 0. **Changes direction:** v changes sign. **Maximum velocity:** a = 0.

WORKED EXAMPLE

A particle moves with $v = t^2 - 5t + 4$ m s^{-1}. Find (a) when it is at rest, (b) its displacement and (c) the total distance travelled from t = 0 to t = 5.

1. (a) $(t - 1)(t - 4) = 0 \Rightarrow$ **t = 1 and t = 4** M1 A1
2. $s(t) = t^3/3 - 5t^2/2 + 4t$: $s(1) = 11/6$, $s(4) = -8/3$, $s(5) = -5/6$ M1
3. (b) displacement = **−5/6 m** A1
4. **(c) distance = 11/6 + 27/6 + 11/6 = 49/6 ≈ 8.17 m** M1 A1

COMMON MISTAKE

Integrating straight from 0 to 5 to get the distance. If the particle turns round, split the integral at each turning time.

LINKING QUESTION

Accelerations come from forces in Newton's laws of motion; the calculus is from Differentiation and Integration; and graphs of motion link to Kinematics in Physics.

Test yourself — cover the answers first

QUICK CHECK

1. What does the area under a v–t graph give?
2. Which suvat equation has no t?
3. A stone is dropped from rest. Find its speed after 3 s.
4. $s = t^3 - 6t^2$. Find a when t = 2.
5. Can you use suvat when a = 2t?

EXAM-STYLE

6. A particle is projected vertically upwards at 25 m s^{-1}. Find the time for which it is at least 20 m above the start.
7. $v = 6t - t^2$ for $0 \le t \le 6$. Find the maximum velocity and the distance travelled.
8. A car decelerates uniformly from 20 m s^{-1} to 8 m s^{-1} over 70 m. Find the deceleration and the time taken.

Show answers

1. displacement
2. $v^2 = u^2 + 2as$
3. 30 m s^{-1}
4. $a = 6t - 12 = 0$
5. no, a must be constant
6. $20 = 25t - 5t^2 \Rightarrow t = 1, 4$; so 3 s
7. $a = 6 - 2t = 0$ at $t = 3$: $v = 9$ m s^{-1}; $\int_0^6 (6t - t^2)\,dt = 108 - 72 = 36$ m
8. $64 = 400 - 140a \Rightarrow a = 2.4$ m s^{-2}; $t = 12/2.4 = 5$ s

Stuck on a question?

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