A LEVEL MATHS 9709 · PAPER 6 PROBABILITY & STATISTICS 2 · 6.2 · ~4 MIN READ

Linear combinations of random variables

Expectation and variance of aX + b and of sums and differences of independent random variables, the difference between 3X and $X_1 + X_2 + X_3$, and the facts that sums of independent normal (or Poisson) variables are normal (or Poisson).

E(aX + b) · Var(aX + b) · E(X ± Y) · Var(X ± Y) · 3X vs $X_1 + X_2 + X_3$ · Sums of normals · Sums of Poissons · Comparing two quantities

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Expectations always add:** E(aX + bY + c) = aE(X) + bE(Y) + c.
- **Variances add for independent variables, even for a difference:** Var(X − Y) = Var(X) + Var(Y). Constants multiply variance by their square.
- **3X is not $X_1 + X_2 + X_3$.** Var(3X) = 9Var(X) (one value tripled); $Var(X_1 + X_2 + X_3) = 3Var(X)$ (three independent values).
- **Normal + normal is normal; Poisson + Poisson is Poisson** (when independent). Sums of Poissons add their λ.

BY THE END YOU CAN

- Use E(aX + b) = aE(X) + b and $Var(aX + b) = a^2Var(X)$.
- Use E and Var for sums and differences of independent variables.
- Use the facts that linear combinations of independent normal variables are normal, and sums of independent Poisson variables are Poisson.
- Solve problems such as P(X > Y) or P(total > k).

1 Expectation and variance

The rules

$$E(aX + b) = aE(X) + b \; ; \; Var(aX + b) = a^2Var(X)$$ FORMULA LIST (MF19)

the constant b shifts but doesn't spread

$$E(aX \pm bY) = aE(X) \pm bE(Y) \; ; \; Var(aX \pm bY) = a^2Var(X) + b^2Var(Y)$$ FORMULA LIST (MF19)

X and Y independent for the variance rule

WORKED EXAMPLE

E(X) = 4, Var(X) = 2, E(Y) = 1, Var(Y) = 3, with X and Y independent. Find E(3X − 2Y + 1) and Var(3X − 2Y + 1).

1. E = 12 − 2 + 1 = **11** B1
2. **Var = 9 × 2 + 4 × 3 = 30** M1 A1

COMMON MISTAKE

Var(X − Y) = Var(X) − Var(Y). Variances never subtract. This is still the commonest error in this topic.

2 Normal combinations

P(X > Y) and totals

WORKED EXAMPLE

X ~ N(20, 9) and Y ~ N(15, 4) are independent. Find P(X > Y).

1. X − Y ~ N(5, 13) M1 A1
2. P(X − Y > 0) = Φ(5/√13) = Φ(1.387) M1
3. **= 0.917** A1

WORKED EXAMPLE

Apple masses are N(150, 10^2) grams. Find the probability that a bag of 6 random apples has total mass over 930 g.

1. T = X_1 + ... + X_6 ~ N(900, 6 × 100 = 600) M1 A1
2. z = 30/√600 = 1.225 M1
3. **1 − 0.8897 = 0.110** A1

- **Compare:** six times one apple's mass, 6X, would have variance 36 × 100 = 3600, which is the wrong model for six different apples.

WORKED EXAMPLE

A ~ N(50, 16) and B ~ N(24, 9) are independent. Find P(A > 2B).

1. A − 2B ~ N(50 − 48, 16 + 4 × 9) = N(2, 52) M1 A1
2. **Φ(2/√52) = Φ(0.277) = 0.609** A1

3 Poisson sums

λ values add

WORKED EXAMPLE

Accidents at junction A follow Po(1.2) per week and at B, Po(0.8) per week, independently. Find the probability of exactly 3 accidents in total in a week.

1. total ~ Po(2) **B1**
2. $e^{-2} \times 2^3/3!$ **M1**
3. **= 0.180** **A1**

COMMON MISTAKE

Adding Poissons that aren't independent, or forming 2X for "two weeks" correctly as Po(2λ) but then 2X for a difference. A difference of Poissons is not Poisson.

LINKING QUESTION

Normal probabilities are in The normal distribution; the Poisson is in The Poisson distribution; and the distribution of a sample mean is in Sampling and estimation.

Test yourself — cover the answers first

QUICK CHECK

1. Var(X) = 5. Find Var(2X + 7).
2. Var(X) = 5, Var(Y) = 3, independent. Find Var(X − Y).
3. X ~ Po(2), Y ~ Po(3), independent. What is X + Y?
4. E(X) = 10. Find E(4 − 3X).
5. Var(X) = 4. Find $\text{Var}(X_1 + X_2)$ and Var(2X).

EXAM-STYLE

6. Weights of men ~ N(80, 100) and women ~ N(65, 64). A lift holds 4 men and 3 women. Find P(total > 550).
7. X ~ N(30, 4). Find $P(X_1 + X_2 > 3X_3 - 25)$.
8. E(X) = 3, Var(X) = 1. Find $E(X^2)$.

Show answers

1. 20
2. 8
3. Po(5)
4. −26
5. 8 and 16
6. N(515, 400 + 192 = 592): z = 35/24.33 = 1.438 ⇒ 0.075
7. $X_1 + X_2 - 3X_3$ ~ N(−30, 4 + 4 + 36 = 44): P(> −25) = 1 − Φ(5/6.633) = 0.225
8. Var + mean² = 1 + 9 = 10

Stuck on a question?

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Want regular help? Send this to your parent →

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