A LEVEL MATHS 9709 · PAPER 5 PROBABILITY & STATISTICS 1 · 5.2 · ~4 MIN READ

Permutations and combinations

Counting arrangements with factorials, repeated items, items kept together or apart, and selections with nCr, including "at least" conditions and probabilities found by counting.

Factorials | Arrangements | Repeated letters | Together & apart | Permutations ${}^nP_r$ | Combinations ${}^nC_r$ | Restrictions | "At least"

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Arrangements of n different items: n!.** With repeats, divide by the factorial of each repeat count.
- **Order matters → permutations ${}^nP_r$; order doesn't matter → combinations ${}^nC_r$.**
- **Together:** glue the group into one block, arrange the blocks, then arrange inside the block. **Apart:** arrange the others first, then place the separated items in the gaps.
- **"At least" questions:** add the separate cases, or subtract the unwanted cases from the total.

BY THE END YOU CAN

- Count arrangements of objects in a line, including repeated objects.
- Solve problems with restrictions: items together, apart, or in fixed positions.
- Count selections using combinations, including with restrictions.
- Find probabilities by counting arrangements or selections.

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1 Arrangements

Factorials and repeats

n different items: n! ; with repeats: n!/(p! q! …) ; ${}^{n}P_{r} = n!/(n - r)!$

${}^{n}P_{r}$ is r items chosen from n, in order

WORKED EXAMPLE

How many arrangements are there of the letters of BANANA?

1. 6 letters, with A three times and N twice M1
2. **6!/(3! 2!) = 60** A1

WORKED EXAMPLE

The 7 letters of ORANGES are arranged. Find the number of arrangements in which (a) the vowels are together and (b) no two vowels are next to each other.

1. (a) block (OAE) + R, N, G, S = 5 items: 5! × 3! = **720** M1 A1
2. (b) arrange the consonants: 4! = 24; 5 gaps, place 3 vowels in order: ${}^{5}P_{3} = 60$ M1
3. **24 × 60 = 1440** A1

- **Fixed positions:** deal with the restricted positions first. "Begins and ends with S" fixes those two places.

COMMON MISTAKE

"Not together" = total – together only works for two items. For three vowels, "no two together" is not "not all three together". The 2024 Paper 5 report noted this confusion.

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2 Selections

Combinations

$$^{n}C_{r} = n!/[r!(n-r)!]$$

the number of ways to choose r from n when order doesn't matter

WORKED EXAMPLE

A committee of 5 is chosen from 6 men and 4 women. How many committees contain at least 2 women?

1. total $^{10}C_5 = 252$ M1
2. no women: $^{6}C_5 = 6$; one woman: $4 \times {}^{6}C_4 = 60$ M1
3. **252 − 6 − 60 = 186** A1

WORKED EXAMPLE

In the committee question, two particular people refuse to serve together. How many committees are possible (no other condition)?

1. committees with both: $^{8}C_3 = 56$ M1
2. **252 − 56 = 196** A1

- **Probability by counting:** P = (number of favourable selections)/(total selections). For a random committee, P(at least 2 women) = 186/252 = 0.738.

COMMON MISTAKE

Multiplying cases that should be added. "2 women or 3 women" are separate cases: add them. Multiply only for choices made together.

LINKING QUESTION

Combinations give the binomial coefficients in Discrete random variables and the expansion in Series; probabilities by counting are in Probability.

Test yourself — cover the answers first

QUICK CHECK

1. Evaluate $^{7}C_3$.
2. How many arrangements of 5 different books on a shelf?
3. How many arrangements of the letters of LEVEL?
4. Choose a captain and vice-captain from 11 players.
5. Choose 3 people from 8 (order doesn't matter).

EXAM-STYLE

6. How many arrangements of STATISTICS are there?
7. 4 boys and 3 girls stand in a line. How many ways if the girls are all together?
8. A team of 4 is chosen from 5 Year 12 and 6 Year 13 students. Find the probability it has exactly 2 from each year.

Show answers

1. 35
2. 120
3. 5!/(2! 2!) = 30
4. $^{11}P_2 = 110$
5. $^{8}C_3 = 56$
6. 10!/(3! 3! 2!) = 50 400
7. 5! × 3! = 720
8. $^{5}C_2 \times {}^{6}C_2/{}^{11}C_4 = 150/330 = 0.455$

Stuck on a question?

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Want regular help? Send this to your parent →

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Ask Riyaz Sir