A LEVEL MATHS 9709 · PAPER 5 PROBABILITY & STATISTICS 1 · 5.3 · ~4 MIN READ

# Probability

Events, the addition rule, mutually exclusive and independent events, conditional probability, tree diagrams with and without replacement, and reversing a tree to find a "given" probability.

Sample spaces | Addition rule | Mutually exclusive | Independent events | Conditional probability | Tree diagrams | Without replacement | Reversed conditioning

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

## IN 30 SECONDS

- **Addition rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. Mutually exclusive events have $P(A \cap B) = 0$.
- **Independent events:** $P(A \cap B) = P(A) \times P(B)$. To show independence, test this equation with numbers.
- **Conditional probability:** $P(A \mid B) = P(A \cap B)/P(B)$. "Given B" shrinks the sample space to B.
- **Trees:** multiply along branches, add the end results you want. Without replacement, the second-stage probabilities change.

## BY THE END YOU CAN

- Use the addition rule and identify mutually exclusive events.
- Test for and use independence.
- Use tree diagrams, including problems without replacement.
- Calculate conditional probabilities, including the reverse of a tree.

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## 1 Combining events

### The rules

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)\ ;\ P(A \mid B) = P(A \cap B)/P(B)$$ FORMULA LIST (MF19)

$$\text{independent} \iff P(A \cap B) = P(A)P(B)$$

- **Mutually exclusive vs independent:** different ideas. Mutually exclusive events can't both happen; independent events don't affect each other. (Two events with non-zero probabilities can't be both.)
- **Venn diagrams** help with two or three events; fill in the intersection first.

WORKED EXAMPLE

**$P(A) = 0.4$, $P(B) = 0.5$ and $P(A \cup B) = 0.7$. Show that A and B are independent, and find $P(A \mid B)$.**

1. $P(A \cap B) = 0.4 + 0.5 - 0.7 = 0.2$ M1
2. $P(A)P(B) = 0.2 = P(A \cap B)$, so independent A1
3. **$P(A \mid B) = 0.2/0.5 = 0.4$ (= P(A), as expected)** B1

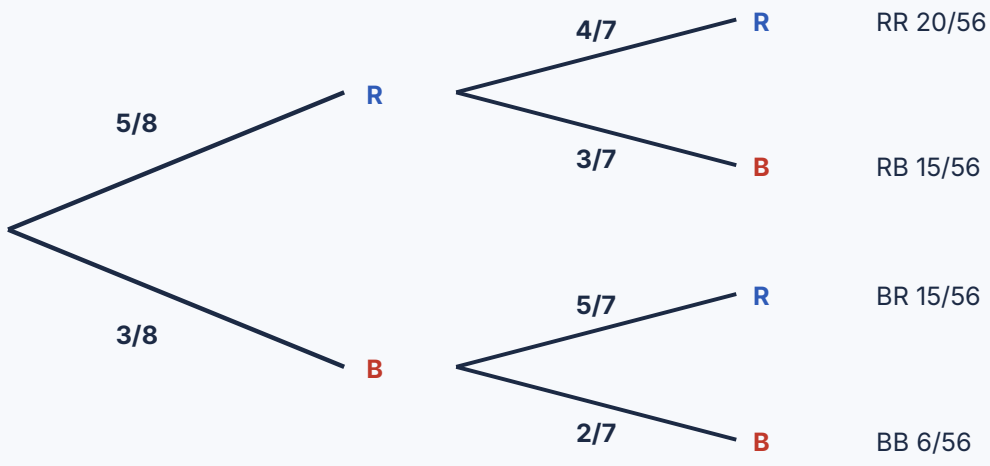
COMMON MISTAKE

**"Independent because they don't overlap".** That describes mutually exclusive events. Always justify independence with numbers.

## 2 Tree diagrams

### With and without replacement

**5 red, 3 blue: two picks without replacement**




Multiply along each branch; the end probabilities add to 1.

WORKED EXAMPLE

**A bag holds 5 red and 3 blue counters. Two are taken without replacement. Find (a) P(same colour) and (b) P(first is red | same colour).**

1. (a) (5/8)(4/7) + (3/8)(2/7) = 20/56 + 6/56 M1
2. = **13/28** A1
3. (b) P(RR)/P(same) = (20/56)/(26/56) M1
4. **= 10/13** A1

WORKED EXAMPLE

**Machine A makes 60% of a factory's items and 3% of its items are faulty; machine B makes the rest and 5% of its items are faulty. An item is chosen at random and is faulty. Find the probability it came from A.**

1. P(faulty) = 0.6 × 0.03 + 0.4 × 0.05 = 0.038 M1 A1
2. P(A | faulty) = 0.018/0.038 M1
3. **= 0.474** A1

COMMON MISTAKE

**Using the branch value 0.03 as P(A | faulty).** That is P(faulty | A). For the reverse, divide the joint probability by the total. This was the most common error on conditional probability in 2024.

LINKING QUESTION

Counting methods are in Permutations and combinations; repeated independent trials lead to the binomial and geometric distributions in Discrete random variables.

## Test yourself — cover the answers first

QUICK CHECK

1. P(A) = 0.3. Find P(A′).
2. A and B are mutually exclusive with P(A) = 0.2, P(B) = 0.5. Find P(A ∪ B).
3. A and B are independent with P(A) = 0.2, P(B) = 0.5. Find P(A ∩ B).
4. P(A ∩ B) = 0.12, P(B) = 0.4. Find P(A | B).
5. Two fair dice are thrown. Find P(total 7).

EXAM-STYLE

6. P(A) = 0.6, P(B) = 0.3, P(A ∪ B) = 0.72. Are A and B independent?
7. Two cards are drawn without replacement from 4 aces and 8 kings. Find P(at least one ace).
8. A die is thrown twice. Given the total is 8, find P(at least one 6).

Show answers

1. 0.7
2. 0.7
3. 0.1
4. 0.3
5. 6/36 = 1/6
6. $P(A \cap B) = 0.18 = 0.6 \times 0.3$, so yes
7. $1 - (8/12)(7/11) = 1 - 14/33 = 19/33$
8. totals of 8: (2,6), (3,5), (4,4), (5,3), (6,2); with a 6: 2, so 2/5


**Stuck on a question?**

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Ask Riyaz Sir