A LEVEL MATHS 9709 · PAPER 5 PROBABILITY & STATISTICS 1 · 5.5 · ~4 MIN READ

The normal distribution

The shape and properties of the normal distribution, standardising with $z = (x - \mu)/\sigma$, using tables forwards and backwards, finding unknown μ and σ, and the normal approximation to the binomial with a continuity correction.

Normal curve | Standardising | Φ(z) tables | Inverse normal | Finding μ and σ | Symmetry | Normal approximation to binomial | Continuity correction

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Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

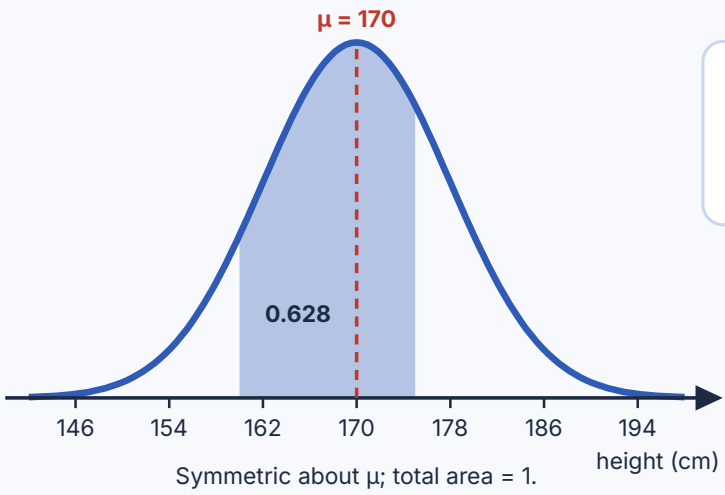
- **$X \sim N(\mu, \sigma^2)$:** symmetric, bell-shaped, with about 68% of values within 1σ of the mean and 95% within 2σ.
- **Standardise:** $z = (x - \mu)/\sigma$ turns any normal into $Z \sim N(0, 1)$, whose cumulative probabilities Φ(z) are in the tables.
- **Working backwards:** from a probability, find z in the tables, then solve $(x - \mu)/\sigma = z$.
- **Approximating B(n, p):** use $N(np, np(1 - p))$ when $np > 5$ and $n(1 - p) > 5$, with a continuity correction of 0.5.

BY THE END YOU CAN

- Use the normal distribution and its symmetry, with tables of Φ(z).
- Find probabilities, and values of x from given probabilities.
- Find unknown μ and/or σ from given probabilities, using simultaneous equations.
- Use the normal approximation to the binomial, with a continuity correction, and know when it is suitable.

1 Probabilities

Standardising



$X \sim N(170, 8^2)$
$P(160 < X < 175)$
$= 0.628$
68% within $\mu \pm \sigma$

Symmetric about μ; total area 1. Probabilities are areas under the curve.

$$Z = (X - \mu)/\sigma \; ; \; P(X < x) = \Phi[(x - \mu)/\sigma] \; ; \; \Phi(-z) = 1 - \Phi(z)$$

tables give $\Phi(z)$ for $z \geq 0$

- **Sketch every time:** shade the area you want, so you can see whether to subtract from 1.
- **N(μ, σ²) has variance σ².** N(50, 16) has σ = 4, not 16.

WORKED EXAMPLE

X ~ N(50, 4²). Find (a) P(X < 56), (b) P(45 < X < 53).

1. (a) $z = (56 - 50)/4 = 1.5$; $\Phi(1.5) =$ **0.933** M1 A1
2. (b) $z = -1.25$ and 0.75: $\Phi(0.75) - [1 - \Phi(1.25)]$ M1
3. **= 0.7734 − 0.1056 = 0.668** A1

2 Working backwards

Inverse normal

WORKED EXAMPLE

X ~ N(50, 4²). Find k such that P(X > k) = 0.2.

1. P(X < k) = 0.8 ⇒ z = 0.842 M1
2. **k = 50 + 0.842 × 4 = 53.4** A1

WORKED EXAMPLE

X ~ N(μ, σ²) with P(X < 20) = 0.1 and P(X > 32) = 0.05. Find μ and σ.

1. $(20 - \mu)/\sigma = -1.282$ and $(32 - \mu)/\sigma = 1.645$ M1 A1 A1
2. subtracting: $12/\sigma = 2.927$ ⇒ **σ = 4.10** M1
3. **μ = 20 + 1.282 × 4.10 = 25.3** A1

COMMON MISTAKE

Signs on z. A value below the mean has a negative z. The 2024 Paper 5 report noted many candidates writing +1.282 in the first equation.

3 Normal approximation to the binomial

B(n, p) ≈ N(np, npq)

- **Continuity correction:** a whole number r covers the interval r − 0.5 to r + 0.5. So P(X ≥ 35) becomes P(Y > 34.5), and P(X < 35) becomes P(Y < 34.5).

WORKED EXAMPLE

X ~ B(80, 0.4). Use a suitable approximation to find P(X ≥ 35).

1. np = 32, npq = 19.2; both np, nq > 5 B1
2. P(Y > 34.5) with Y ~ N(32, 19.2) M1
3. z = 2.5/√19.2 = 0.571 ⇒ 1 − 0.716 = **0.284** M1 A1
4. **(the exact binomial value is 0.283)**

COMMON MISTAKE

Missing or wrong-way continuity corrections, or using npq as the standard deviation. Write the inequality in X, then change it.

LINKING QUESTION

The binomial is in Discrete random variables; the normal distribution of sample means is in Sampling and estimation; and normal tests are in Hypothesis tests.

Test yourself — cover the answers first

QUICK CHECK

1. Find Φ(0) for Z ~ N(0, 1).
2. X ~ N(30, 9). What is σ?
3. Write P(Z > −1) in terms of Φ(1).
4. Continuity-correct P(X ≤ 20) for a binomial X.
5. When is the normal approximation to B(n, p) suitable?

EXAM-STYLE

6. Lengths are N(120, 25). Find the probability a length is within 8 of the mean.
7. X ~ N(μ, 36) and P(X > 70) = 0.0228. Find μ.
8. X ~ B(50, 0.5). Use the normal approximation to estimate P(X = 25).

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Show answers

1. 0.5
2. 3
3. $\Phi(1)$
4. $P(Y < 20.5)$
5. $np > 5$ and $n(1 - p) > 5$
6. $z = \pm 1.6$: $2\Phi(1.6) - 1 = 0.890$
7. $z = 2.0 \Rightarrow \mu = 70 - 12 = 58$
8. $N(25, 12.5)$: $P(24.5 < Y < 25.5) = 2\Phi(0.141) - 1 = 0.112$

Stuck on a question?

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