A LEVEL MATHS 9709 · PAPER 6 PROBABILITY & STATISTICS 2 · 6.1 · ~4 MIN READ

The Poisson distribution

Modelling random events with Po(λ), its conditions, scaling λ to a new interval, mean equal to variance, and using the Poisson to approximate the binomial and the normal to approximate the Poisson.

Po(λ) | Conditions | P(X = r) | Scaling λ | Mean = variance | Poisson approximation to binomial | Normal approximation to Poisson | Continuity correction

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Po(λ) counts random events in a fixed interval** of time or space: events happen singly, independently, and at a constant average rate λ.
- **Mean = variance = λ.** If a sample has mean close to its variance, a Poisson model is plausible.
- **Change of interval:** scale λ in proportion. 4 per hour means 2 per half hour.
- **Approximations:** B(n, p) ≈ Po(np) when n is large (n > 50) and np < 5; Po(λ) ≈ N(λ, λ) when λ > 15, with a continuity correction.

BY THE END YOU CAN

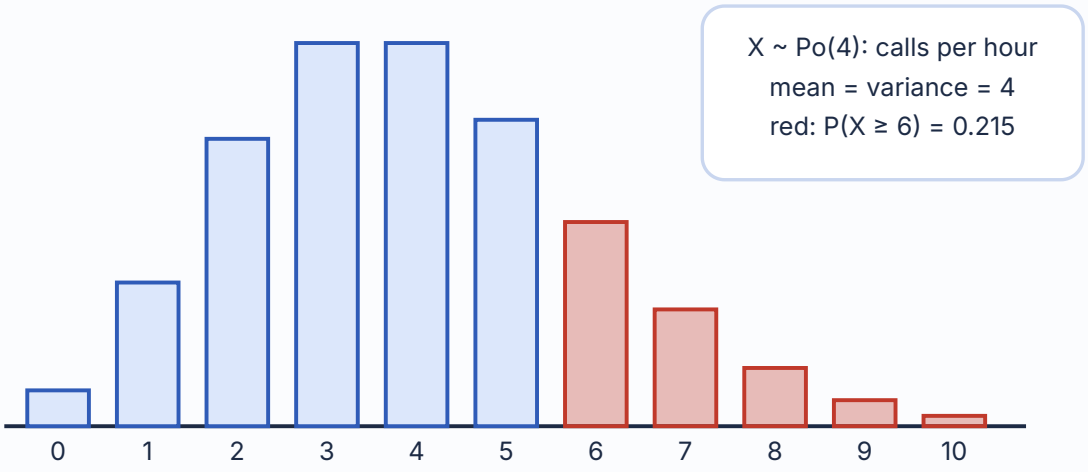
- Use the Poisson distribution as a model and state its conditions in context.
- Calculate Poisson probabilities, including for a scaled interval.
- Use E(X) = Var(X) = λ.
- Use the Poisson approximation to the binomial and the normal approximation to the Poisson.

1 The Poisson model

Po(λ)

$$P(X = r) = e^{-\lambda}\lambda^r/r!\ ,\ r = 0, 1, 2, \ldots ;\ E(X) = Var(X) = \lambda$$ FORMULA LIST (MF19)

λ is the mean number of events in the interval



Po(4): skewed for small λ, and more symmetric as λ grows.

- **Conditions in context:** "the calls occur randomly, independently, one at a time, at a constant average rate". Give them in the context of the question.

WORKED EXAMPLE

X ~ Po(3.2). Find (a) P(X = 2), (b) P(X ≥ 3).

1. (a) $e^{-3.2} \times 3.2^2/2 =$ **0.209** M1 A1
2. (b) $1 - [P(0) + P(1) + P(2)] = 1 - e^{-3.2}(1 + 3.2 + 5.12)$ M1
3. **= 0.620** A1

WORKED EXAMPLE

Calls arrive at an average rate of 4 per hour. Find the probability of at most 1 call in a 30-minute period.

1. λ = 2 for 30 minutes B1
2. $e^{-2}(1 + 2) = 3e^{-2}$ M1
3. **= 0.406** A1

COMMON MISTAKE

Not scaling λ, or rescaling the probability instead of λ. The 2024 Paper 6 report also asked for conditions "in context": a bare "independent" scores nothing.

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2 Approximations

Poisson for binomial; normal for Poisson

- **Poisson for binomial:** many trials with a small success probability. Use $\lambda = np$.
- **Normal for Poisson:** $\lambda > 15$, $Y \sim N(\lambda, \lambda)$, and continuity correction because X is discrete.

WORKED EXAMPLE

$X \sim B(200, 0.015)$. Use a suitable approximation to find $P(X \leq 2)$.

1. $n > 50$, $np = 3 < 5$, so use Po(3) B1
2. $e^{-3}(1 + 3 + 4.5)$ M1
3. **$= 0.423$** A1 **(exact binomial: 0.421)**

WORKED EXAMPLE

The number of emails a day is Po(25). Use a suitable approximation to find $P(X > 30)$.

1. $\lambda > 15$: $Y \sim N(25, 25)$ B1
2. $P(Y > 30.5)$: $z = 5.5/5 = 1.1$ M1
3. **$1 - 0.8643 = 0.136$** A1

COMMON MISTAKE

Using $\sigma = 25$ instead of $\sqrt{25} = 5$, or forgetting the continuity correction.

LINKING QUESTION

The binomial is in Discrete random variables; sums of Poisson variables are in Linear combinations; and Poisson hypothesis tests are in Hypothesis tests.

Test yourself — cover the answers first

QUICK CHECK

1. $X \sim Po(5)$. Find Var(X).
2. $X \sim Po(2)$. Find $P(X = 0)$.
3. A machine averages 6 faults per week. What is λ for 2 weeks?
4. When can B(n, p) be approximated by a Poisson?
5. Continuity-correct $P(X \geq 20)$ for a normal approximation.

EXAM-STYLE

6. $X \sim Po(1.5)$. Find $P(X \leq 2 \mid X \geq 1)$.
7. Typing errors occur at 0.8 per page. Find the probability of more than 3 errors in 5 pages.
8. Sample data: mean 3.1, variance 7.2. Is a Poisson model suitable?

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Show answers

1. 5
2. $e^{-2} = 0.135$
3. 12
4. n large (> 50) and np < 5
5. P(Y > 19.5)
6. [P(1) + P(2)]/[1 − P(0)] = (0.3347 + 0.2510)/0.7769 = 0.754
7. λ = 4: $1 - e^{-4}(1 + 4 + 8 + 10.667) = 0.567$
8. no, the variance is much larger than the mean

Stuck on a question?

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Want regular help? Send this to your parent →

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