A LEVEL MATHS 9709 · PAPER 3 PURE MATHEMATICS 3 · 3.7 · ~6 MIN READ

Vectors

Vectors in two and three dimensions, magnitude and unit vectors, the scalar product and angles, the vector equation of a line, and deciding whether two lines are parallel, intersect or are skew, with the foot of a perpendicular and shortest distances.

Position & direction vectors | Magnitude | Unit vectors | Scalar product | Angle between vectors | Vector equation of a line | Parallel, intersecting, skew | Foot of perpendicular

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** FORMULA LIST (MF19) given in the exam M1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **AB = b − a** (end minus start). Its length is |AB| = √(x² + y² + z²).
- **Scalar product:** a·b = $a_1b_1 + a_2b_2 + a_3b_3$ = |a||b| cos θ. Zero means perpendicular.
- **A line is r = a + tb:** a point on it plus any multiple of its direction.
- **Two lines in 3D** are parallel (directions are multiples), intersect (a single solution for s and t), or are skew (neither).

BY THE END YOU CAN

- Use vector notation in 2D and 3D: column vectors, i, j, k, position vectors and magnitudes.
- Use the scalar product to find angles and test for perpendicularity.
- Form the vector equation of a line through two points, or through a point with a given direction.
- Decide whether two lines are parallel, intersect or are skew, and find the point of intersection.
- Find the foot of the perpendicular from a point to a line and the shortest distance.

Riyaz Sir

1 Vectors and the scalar product

Basics

- **Notation:** {(3;−2;2)} = 3i − 2j + 2k. A **position vector** OA (written a) goes from the origin to A.
- **Unit vector** in the direction of a: a/|a|. **Collinear** points: AB is a multiple of BC.

$$\mathbf{a}\cdot\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\mathbf{a}|\,|\mathbf{b}|\cos\theta$$ FORMULA LIST (MF19)

$\mathbf{a}\cdot\mathbf{b} = 0 \iff$ perpendicular (non-zero vectors)

WORKED EXAMPLE

Find the angle between a = i + 2j + 2k and b = 2i − j + 2k.

1. a·b = 2 − 2 + 4 = 4; |a| = 3, |b| = 3 M1
2. cos θ = 4/9 M1
3. **θ = 63.6°** A1

- **Angle at a vertex:** for angle ABC use BA and BC, both starting at B. Using AB and BC gives the supplementary angle.

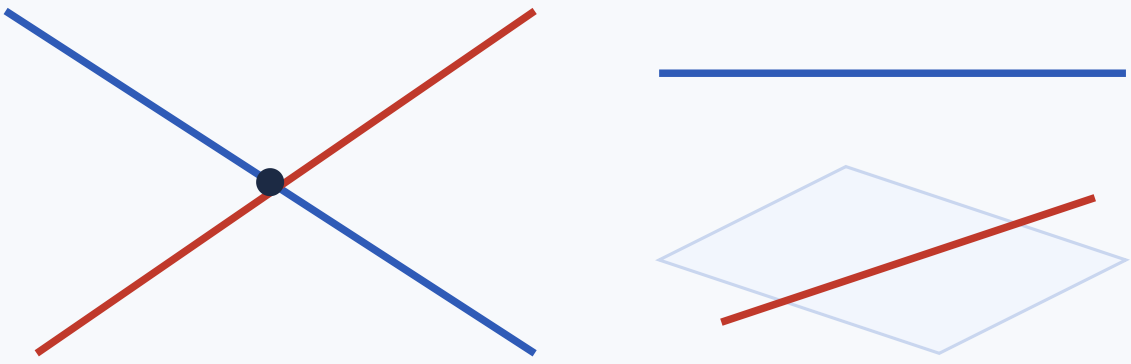
COMMON MISTAKE

Vectors pointing the wrong way. For angle ABC, both vectors must point away from B.

2 Lines

r = a + tb

Lines in 3D: parallel, intersecting, or skew



intersecting **skew: not parallel, never meet**

A line in 3D: a fixed point, and a direction you can travel along any distance.

- **Through A and B:** r = a + t(b − a). Any point on the line has coordinates written in terms of t.
- **Different-looking equations can describe the same line:** a different point or a multiple of the direction.

WORKED EXAMPLE

Find a vector equation of the line through A(1, 2, 3) and B(4, 0, 5).

1. AB = b − a = 3i − 2j + 2k **B1**
2. **r = i + 2j + 3k + t(3i − 2j + 2k)** **B1**

Parallel, intersecting or skew

- **Step 1:** are the directions multiples of each other? Yes → parallel (or the same line).
- **Step 2:** set the x, y, z components equal. Solve two equations for s and t; **check the third**. Consistent → they intersect; inconsistent → skew.

WORKED EXAMPLE

l_1: r = (1, 0, 2) + s(1, 1, −1) and l_2: r = (0, −3, 1) + t(1, 2, 0). Show they intersect, and find the point and the angle between them.

1. 1 + s = t, s = −3 + 2t, 2 − s = 1 **M1**
2. third equation: s = 1; then t = 2, check s = −3 + 4 = 1 ✓ **A1**
3. point **(2, 1, 1)** **A1**
4. **directions: (1·1 + 1·2 + 0)/(√3 √5) = 3/√15 ⇒ θ = 39.2°** **M1** **A1**

WORKED EXAMPLE

Show that l_1 above and l_3: r = (2, 3, −1) + t(0, 1, 1) are skew.

1. directions (1, 1, −1) and (0, 1, 1) are not multiples, so not parallel **B1**
2. 1 + s = 2 ⇒ s = 1; s = 3 + t ⇒ t = −2 **M1**
3. **z: 2 − 1 = 1 but −1 + (−2) = −3; 1 ≠ −3, so no intersection: skew** **A1**

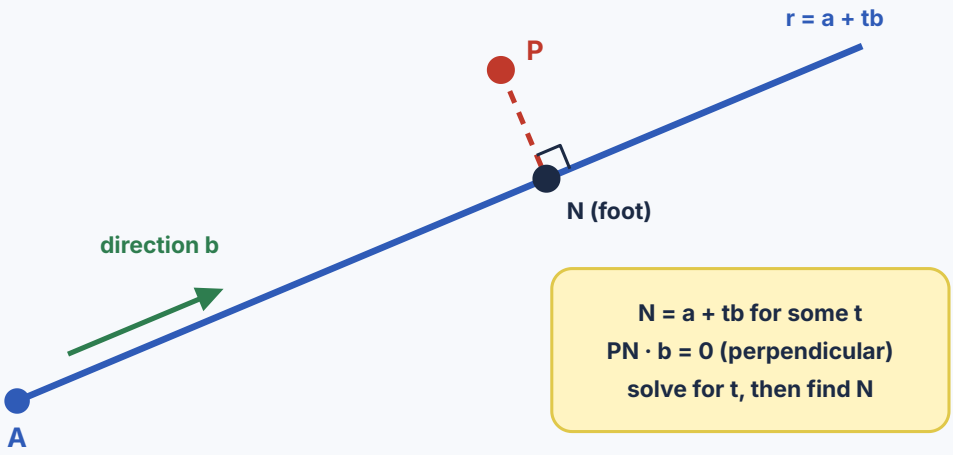
COMMON MISTAKE

Using the same parameter for both lines. Each line needs its own (s and t).

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3 Perpendicular distance

The foot of the perpendicular



The shortest distance from P to the line is PN, where PN is perpendicular to the line.

PN is the shortest distance from P to the line: it meets the line at right angles.

- **Method:** write N = a + tb (a general point), form PN = n − p, then solve PN·b = 0 for t.
- **Shortest distance** = |PN|. You can also find it using sin θ after finding the angle between AP and the line.

WORKED EXAMPLE

Find the foot of the perpendicular from P(6, 0, 2) to the line r = (1, 2, 3) + t(3, −2, 2), and the perpendicular distance.

1. N = (1 + 3t, 2 − 2t, 3 + 2t); PN = (3t − 5, 2 − 2t, 1 + 2t) M1
2. PN·(3, −2, 2) = 9t − 15 − 4 + 4t + 2 + 4t = 17t − 17 = 0 ⇒ t = 1 M1 A1
3. **N = (4, 0, 5), PN = (−2, 0, 3), distance √13 ≈ 3.61** A1

COMMON MISTAKE

Dotting with the wrong vector. PN must be dotted with the direction b, not with the position vector a.

LINKING QUESTION

2D vectors are used for forces in Forces and equilibrium; complex numbers in Complex numbers behave like 2D vectors on an Argand diagram.

Test yourself — cover the answers first

QUICK CHECK

1. Find |2i − j + 2k|.
2. a·b = 0 for non-zero a, b. What does this mean?
3. Write the line through (1, 0, 4) parallel to 2i + k.
4. Two lines are not parallel and do not meet. What are they?

5. For angle ABC, which two vectors do you use?

EXAM-STYLE

6. The points A(2, −1, 3), B(5, 1, 2), C(3, 4, k). Find k so that AB is perpendicular to BC.
7. Find the unit vector in the direction of 4i − 4j + 2k.
8. Show that the point (7, −2, 7) lies on r = (1, 2, 3) + t(3, −2, 2).

Show answers

1. 3
2. a and b are perpendicular
3. r = i + 4k + t(2i + k)
4. skew
5. BA and BC
6. AB = (3, 2, −1), BC = (−2, 3, k − 2); −6 + 6 − (k − 2) = 0 ⇒ k = 2
7. |v| = 6, so (2/3)i − (2/3)j + (1/3)k
8. t = 2 gives (7, −2, 7) in all three components

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

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