A LEVEL PHYSICS 9702 · A LEVEL · TOPIC 19 · ~4 MIN READ

Capacitance

Capacitance C = Q/V, capacitors in series and parallel, the energy stored as ½QV, the capacitance of an isolated sphere, and the exponential discharge of a capacitor through a resistor with time constant RC.

Capacitance C = Q/V | Isolated sphere $4\pi\varepsilon_o r$ | Series & parallel | Energy W = ½QV = ½CV² | Area under Q–V graph | Discharge Q = Q₀e^{−t/RC} | Time constant τ = RC

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Paper tag = **where it is examined** DATA & FORMULAE given in the exam C1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Capacitance C = Q/V:** the charge stored per unit potential difference. Unit: farad (F) = C V^{-1}.
- **Parallel:** $C = C_1 + C_2 + \ldots$. **Series:** $1/C = 1/C_1 + 1/C_2 + \ldots$. (The opposite of resistors.)
- **Energy stored W = ½QV = ½CV²:** the area under a graph of V against Q.
- **Discharge through R:** Q, V and I all fall exponentially: $x = x_o e^{-t/RC}$. The time constant τ = RC is the time to fall to 1/e (37%).

BY THE END YOU CAN

- Define capacitance and use C = Q/V, including for an isolated sphere $C = 4\pi\varepsilon_o r$.
- Derive and use the formulae for capacitors in series and in parallel.
- Determine the energy stored from the area under a potential–charge graph, and use W = ½QV = ½CV².
- Analyse discharge using $x = x_o e^{-t/RC}$ and the time constant τ = RC.

1 Capacitance

C = Q/V

Capacitance — the charge stored per unit potential difference: C = Q/V.

- **Parallel-plate capacitor:** equal and opposite charges +Q and −Q on the plates; the net charge is zero.
- **Isolated sphere:** $V = Q/(4\pi\varepsilon_o r)$, so $C = 4\pi\varepsilon_o r$. A sphere of radius 0.10 m has $C = 1.1 \times 10^{-11}$ F.

Parallel: $C = C_1 + C_2 + \ldots$; Series: $1/C = 1/C_1 + 1/C_2 + \ldots$ DATA & FORMULAE

parallel share V; series share Q

WORKED EXAMPLE

Find the total capacitance of a 2 μF and a 3 μF in series, with a 4 μF in parallel with that pair.

1. series: 1/C = 1/2 + 1/3 ⇒ C = 1.2 μF C1
2. **parallel: C = 1.2 + 4 = 5.2 μF** A1

COMMON MISTAKE

Using the resistor rules for capacitors. Capacitors in parallel add directly.

2 Energy stored

W = ½QV

$$W = \tfrac{1}{2}QV = \tfrac{1}{2}CV^2 = Q^2/(2C)$$ DATA & FORMULAE

area under the V–Q graph (a triangle)

- **Why the ½:** as charge builds up, the p.d. rises from 0 to V, so the average p.d. while charging is V/2.

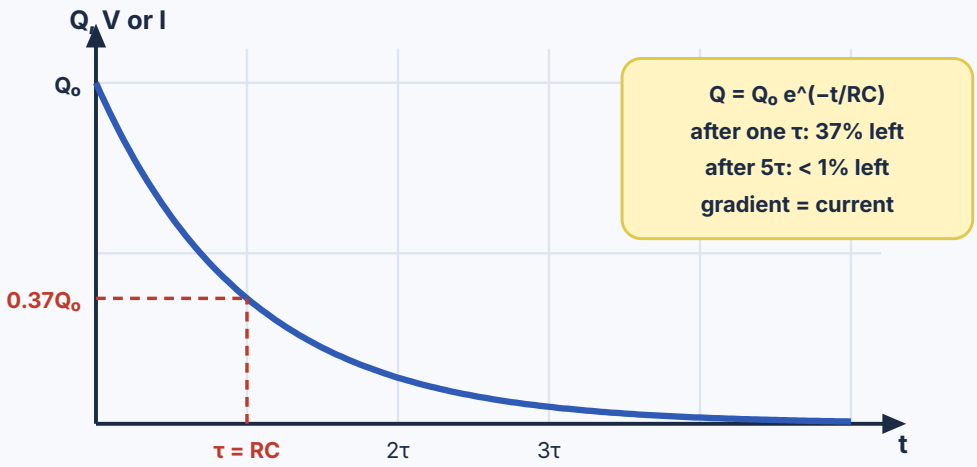
WORKED EXAMPLE

A 470 μF capacitor is charged to 12 V. Find the charge and the energy stored.

1. $Q = CV = 470 \times 10^{-6} \times 12 =$ **5.6×10^{-3} C** A1
2. **$W = \tfrac{1}{2}CV^2 = \tfrac{1}{2} \times 470 \times 10^{-6} \times 144 = 0.034$ J** A1

3 Discharge

Exponential decay



Discharge through a resistor: the same exponential shape for Q, V and I.

Discharge: Q, V and I fall by the same fraction in each equal time interval.

$$x = x_0 e^{-t/RC} ; \tau = RC$$ DATA & FORMULAE

x stands for Q, V or I

WORKED EXAMPLE

The 470 μF capacitor (12 V) discharges through 10 kΩ. Find (a) the time constant, (b) the p.d. after 3.0 s, (c) the time for the p.d. to fall to 3.0 V.

1. (a) $\tau = RC = 10 \times 10^{3} \times 470 \times 10^{-6} =$ **4.7 s** A1
2. (b) $V = 12e^{-3.0/4.7} =$ **6.3 V** C1 A1
3. **(c) $3 = 12e^{-t/4.7} \Rightarrow t = 4.7 \ln 4 = 6.5$ s** C1 A1

- **Finding RC from a graph:** plot ln V against t; the gradient is −1/RC.
- **Charging** (not needed in detail): Q rises towards CE with the same time constant.

COMMON MISTAKE

Taking logs of the wrong side, or forgetting that $\ln(V/V_o)$ is negative. Rearrange to $t = RC \ln(V_o/V)$.

LINKING QUESTION

Charge and current are in Electricity; potential from a point charge is in Electric fields; and exponential decay reappears in Nuclear physics. Smoothing capacitors are in Alternating currents.

Test yourself — cover the answers first

QUICK CHECK

1. Define the farad.
2. Find C for 3 μF and 6 μF in series.
3. What fraction of the charge remains after one time constant?
4. A capacitor's p.d. doubles. What happens to the energy stored?
5. What does the area under a V–Q graph represent?

EXAM-STYLE

6. A 2200 μF capacitor at 9.0 V discharges through a lamp of 50 Ω. Find the initial current and the charge left after 0.20 s.
7. Find the time for the energy stored to halve during discharge.
8. Three 6 μF capacitors are in series across 12 V. Find the charge on each and the total energy.

Show answers

1. one coulomb per volt
2. 2 μF
3. 1/e ≈ 0.37
4. it quadruples
5. energy stored
6. $I_o = 0.18$ A; $Q = 0.0198\, e^{-0.20/0.11} = 3.2 \times 10^{-3}$ C
7. $W \propto V^2 \propto e^{-2t/RC}$: $t = (RC/2) \ln 2 = 0.35RC$
8. C = 2 μF; Q = 24 μC each; $W = \tfrac{1}{2} \times 2 \times 10^{-6} \times 144 = 1.44 \times 10^{-4}$ J

Stuck on a question?

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