A LEVEL PHYSICS 9702 · A LEVEL · TOPIC 15 · ~4 MIN READ

Ideal gases

The mole and the Avogadro constant, the ideal gas equation pV = nRT = NkT, the kinetic theory assumptions, deriving $pV = \frac{1}{3}Nm\langle c^2\rangle$, and linking the mean kinetic energy of a molecule to temperature.

Amount of substance | Avogadro constant | Ideal gas equation | Boltzmann constant | Kinetic theory assumptions | $pV = \frac{1}{3}Nm\langle c^2\rangle$ | Mean kinetic energy | r.m.s. speed

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** DATA & FORMULAE given in the exam C1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **The mole:** 1 mol contains $N_A = 6.02 \times 10^{23}$ particles.
- **An ideal gas obeys pV = nRT** (n moles) **= NkT** (N molecules), with T in kelvin. $R = 8.31\ J\ K^{-1}\ mol^{-1}$; $k = R/N_A = 1.38 \times 10^{-23}\ J\ K^{-1}$.
- **Kinetic theory:** molecules bouncing off the walls change momentum, giving pressure: $pV = \frac{1}{3}Nm\langle c^2\rangle$.
- **Temperature measures the mean kinetic energy** of the molecules: $\frac{1}{2}m\langle c^2\rangle = (3/2)kT$.

BY THE END YOU CAN

- Understand amount of substance in moles and the Avogadro constant.
- Recall and use pV = nRT and pV = NkT.
- State the basic assumptions of the kinetic theory of gases.
- Derive and use $pV = \frac{1}{3}Nm\langle c^2\rangle$, and link mean kinetic energy to temperature.

1 The ideal gas equation

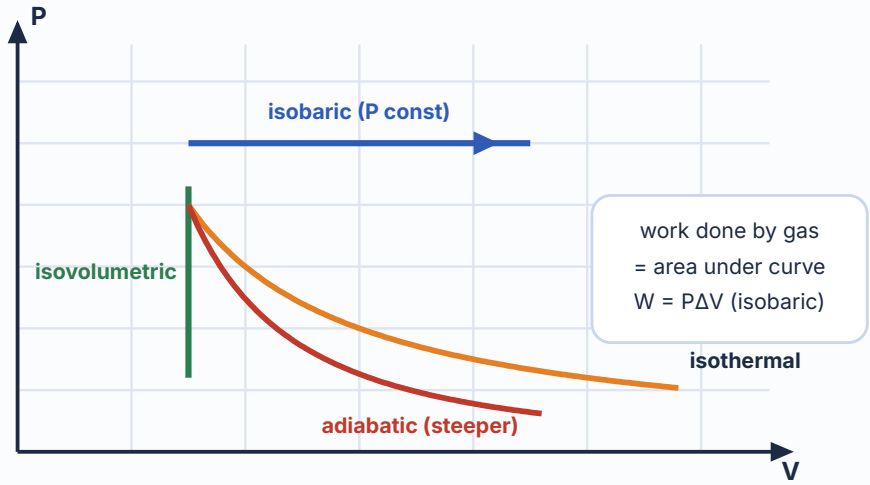
pV = nRT

Ideal gas — a gas that obeys $pV \propto T$ at all pressures, volumes and temperatures (T in kelvin).

$$pV = nRT = NkT\ ;\ k = R/N_A$$

p in Pa, V in m^3, T in K

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At constant temperature, p ∝ 1/V; the hotter isotherm sits further out.

WORKED EXAMPLE

Find the number of moles of gas in 0.024 m³ at 1.0 × 10⁵ Pa and 290 K.

1. n = pV/RT = 1.0 × 10⁵ × 0.024/(8.31 × 290) C1
2. **n = 1.0 mol** A1

WORKED EXAMPLE

A sealed rigid container of gas is heated from 300 K to 375 K. By what factor does the pressure change?

1. constant V: p/T is constant C1
2. **$p_2 = p_1 \times 375/300 = 1.25\ p_1$** A1

COMMON MISTAKE

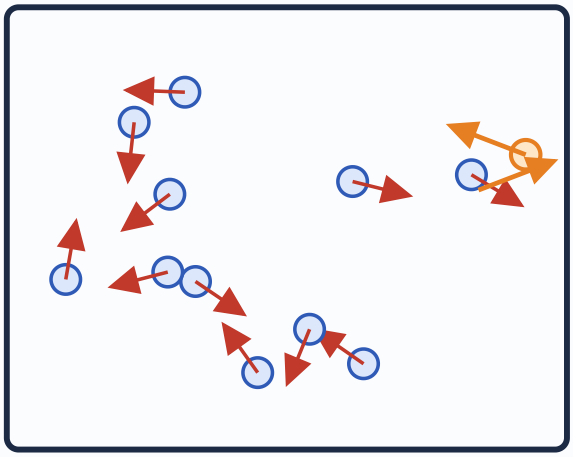
Using litres or cm³ and °C. Convert: 1 dm³ = 10⁻³ m³, 1 cm³ = 10⁻⁶ m³, and T in kelvin.

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2 Kinetic theory

Assumptions

Assumption	Why it matters
A large number of molecules in random motion	pressure is the same in all directions, and statistics work
Molecules have negligible volume compared with the container	V is the volume available for motion
No intermolecular forces except during collisions	no potential energy: internal energy is all kinetic
Collisions are elastic	no kinetic energy is lost
The time of a collision is negligible compared with the time between collisions	collisions are instantaneous



Each collision with a wall reverses the momentum component: $\Delta p = 2mv$. Many collisions per second give a steady force, so a pressure $P = F/A$.

$P = \frac{1}{3}\rho\langle v^2\rangle$

Each wall collision reverses a molecule's velocity component, changing its momentum by 2mu.

Deriving $pV = \frac{1}{3}Nm\langle c^2\rangle$

- **One molecule in a cube of side L:** it hits a wall with velocity u, changing momentum by 2mu. Time between hits on that wall: 2L/u. Average force = 2mu ÷ (2L/u) = mu^2/L.
- **N molecules:** total force = $Nm\langle u^2\rangle/L$, so pressure = $Nm\langle u^2\rangle/L^3 = Nm\langle u^2\rangle/V$.
- **Three dimensions:** $\langle c^2\rangle = \langle u^2\rangle + \langle v^2\rangle + \langle w^2\rangle = 3\langle u^2\rangle$ (random motion), so $pV = \frac{1}{3}Nm\langle c^2\rangle$.

$$pV = \tfrac{1}{3}Nm\langle c^2\rangle \; ; \; \tfrac{1}{2}m\langle c^2\rangle = (3/2)kT$$ DATA & FORMULAE

combine with $pV = NkT$

WORKED EXAMPLE

Find the mean kinetic energy of a molecule at 300 K, and the r.m.s. speed of nitrogen molecules (mass 28 u).

1. $E = (3/2)kT = 1.5 \times 1.38 \times 10^{-23} \times 300 = \mathbf{6.2 \times 10^{-21}\ J}$ C1 A1
2. $\mathbf{c_{rms} = \sqrt{(3kT/m)} = \sqrt{(3 \times 1.38 \times 10^{-23} \times 300/(28 \times 1.66 \times 10^{-27}))} = 517\ m\ s^{-1}}$ A1

COMMON MISTAKE

Confusing $\langle c^2 \rangle$ with $\langle c \rangle^2$. The r.m.s. speed is $\sqrt{\langle c^2 \rangle}$: square the speeds, find the mean, then square-root.

LINKING QUESTION

Internal energy and the first law are in Thermodynamics; the kelvin scale is in Temperature; momentum change is from Dynamics.

Test yourself — cover the answers first

QUICK CHECK

1. State the Avogadro constant.
2. What happens to the mean KE of molecules if T (in K) doubles?
3. Name one assumption of kinetic theory.
4. Find the r.m.s. speed of speeds 2, 4, 6 m s^{-1}.
5. How many molecules are in 2 mol?

EXAM-STYLE

6. A gas at 1.2×10^5 Pa occupies 3.0×10^{-3} m^3 at 27 °C. Find the number of molecules.
7. By what factor does c_rms change if T rises from 300 K to 1200 K?
8. Show that $p = \frac{1}{3}\rho\langle c^2 \rangle$.

Show answers

1. 6.02×10^{23} mol^{-1}
2. it doubles
3. e.g. collisions are elastic
4. $\sqrt{(56/3)} = 4.32$ m s^{-1}
5. 1.2×10^{24}
6. $N = pV/kT = 360/(1.38 \times 10^{-23} \times 300) = 8.7 \times 10^{22}$
7. $c_{rms} \propto \sqrt{T}$, so it doubles
8. $pV = \frac{1}{3}Nm\langle c^2 \rangle$ and $\rho = Nm/V$, so $p = \frac{1}{3}\rho\langle c^2 \rangle$

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