A LEVEL PHYSICS 9702 · A LEVEL · TOPIC 12 · ~5 MIN READ

Motion in a circle

Radians and angular speed, why uniform circular motion needs a resultant force towards the centre, centripetal acceleration $a = r\omega^2 = v^2/r$, and applying $F = mv^2/r$ to cars on bends, vertical circles and orbits.

Radians | Angular displacement | Angular speed ω | v = rω | Centripetal acceleration | Centripetal force | Vertical circles | Banked & flat bends

RS by Riyaz Sir
mentorriyaz.in

Paper tag = **where it is examined** DATA & FORMULAE given in the exam C1 A1 B1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Angles in radians:** a full turn is 2π rad. Angular speed $\omega = \Delta\theta/\Delta t = 2\pi/T = 2\pi f$.
- **Speed v = rω.** Even at constant speed, the direction changes, so the object accelerates towards the centre.
- **Centripetal acceleration $a = r\omega^2 = v^2/r$; resultant force $F = mv^2/r = mr\omega^2$,** always towards the centre.
- **Centripetal force is not a new force.** It is the resultant of real forces: tension, friction, gravity, a normal reaction.

BY THE END YOU CAN

- Define the radian and express angular displacement in radians.
- Understand and use angular speed ω, and v = rω.
- Explain that a resultant force of constant size perpendicular to the velocity produces uniform circular motion.
- Recall and use $a = r\omega^2 = v^2/r$ and $F = mr\omega^2 = mv^2/r$.

1 Describing circular motion

Radians and angular speed

Radian — the angle subtended at the centre of a circle by an arc equal in length to the radius.

Angular speed ω — the rate of change of angular displacement: $\omega = \Delta\theta/\Delta t$.

$$\omega = 2\pi/T = 2\pi f \; ; \; v = r\omega$$

T = period, f = frequency of rotation

WORKED EXAMPLE

Find the angular speed of the Earth's rotation, and the centripetal acceleration of a point on the equator (radius 6.37×10^6 m).

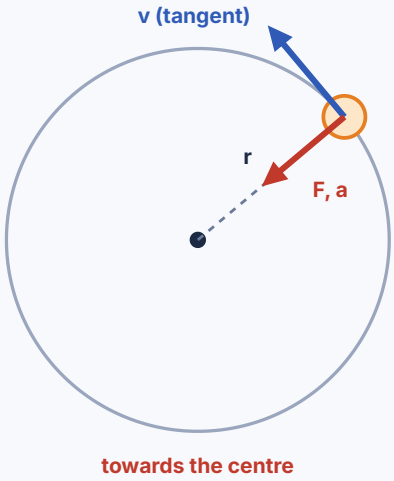
1. $\omega = 2\pi/(24 \times 3600) =$ **7.27×10^{-5} rad s^{-1}** C1 A1
2. **$a = r\omega^2 = 6.37 \times 10^6 \times (7.27 \times 10^{-5})^2 = 0.0337$ m s^{-2}** A1

COMMON MISTAKE

Using degrees, or rpm, in ω. Convert to rad s^{-1} first: 300 rpm = 300 × 2π/60 = 31.4 rad s^{-1}.

2 Centripetal force

Why it accelerates



$v = 2\pi r/T = \omega r$
$a = v^2/r = \omega^2 r = 4\pi^2 r/T^2$
$F = mv^2/r$
The force is at 90° to v, so it changes direction, not speed. No work done. Something must supply it: tension, friction, gravity...

Velocity is along the tangent; acceleration and resultant force point to the centre.

$$a = r\omega^2 = v^2/r \; ; \; F = mr\omega^2 = mv^2/r$$

directed towards the centre of the circle

- **Why no work is done:** the force is perpendicular to the velocity, so the speed (and kinetic energy) stays constant.
- **If the force is removed,** the object moves off along the tangent (Newton's first law), not outwards.

WORKED EXAMPLE

A car of mass 1200 kg travels round a flat bend of radius 50 m at 15 m s^{-1}. Find the friction force needed.

1. $F = mv^2/r = 1200 \times 15^2/50$ C1
2. **F = 5400 N, provided by friction towards the centre** A1

Vertical circles

- **At the top:** $T + mg = mv^2/r$ (both point down, to the centre). **At the bottom:** $T - mg = mv^2/r$. So the tension is greatest at the bottom.
- **Least speed at the top** (string just slack, T = 0): $mg = mv^2/r$, so $v = \sqrt{(gr)}$.

WORKED EXAMPLE

A 0.20 kg mass whirls in a vertical circle on a string of length 0.80 m. (a) Find the least speed at the top for the string to stay taut. (b) At the bottom its speed is 6.0 m s^{-1}; find the tension.

1. (a) $v = \sqrt{(gr)} = \sqrt{(9.81 \times 0.80)} =$ **2.80 m s^{-1}** C1 A1
2. (b) $T - mg = mv^2/r \Rightarrow T = 0.20 \times 9.81 + 0.20 \times 36/0.80$ C1
3. **T = 11.0 N** A1

COMMON MISTAKE

Drawing "centripetal force" as an extra arrow on a free-body diagram. Draw only real forces; their resultant is the centripetal force. This was a common 2024 Paper 4 error.

LINKING QUESTION

Gravity as the centripetal force for orbits is in Gravitational fields; charged particles moving in circles are in Magnetic fields; and SHM is the projection of circular motion: Oscillations.

Test yourself — cover the answers first

QUICK CHECK

1. Convert 90° to radians.
2. A wheel turns at 5 revolutions per second. Find ω.
3. What is the direction of centripetal acceleration?
4. Why does the kinetic energy of an object in uniform circular motion stay constant?
5. Find v for r = 0.5 m and ω = 8 rad s^{-1}.

EXAM-STYLE

6. A 50 g stone on a 0.40 m string moves in a horizontal circle at 3.0 revolutions per second. Find the tension (ignore gravity).
7. Explain why passengers feel lighter at the top of a hump-backed bridge.
8. A satellite orbits at 7.5 km s^{-1} at radius 7.0 × 10^6 m. Find its centripetal acceleration.

Show answers

1. π/2
2. 10π = 31.4 rad s^{-1}
3. towards the centre of the circle
4. the force is perpendicular to the velocity, so it does no work
5. 4 m s^{-1}
6. ω = 6π = 18.85 rad s^{-1}; T = mr$ω^2$ = 0.050 × 0.40 × 355 = 7.1 N
7. mg − R = mv^2/r, so R < mg; the reaction (felt weight) is reduced
8. a = v^2/r = $(7500)^2$/7.0 × 10^6 = 8.0 m s^{-2}

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