A LEVEL PHYSICS 9702 · A LEVEL · TOPIC 17 · ~5 MIN READ

Oscillations

Describing oscillations, simple harmonic motion defined by $a = -\omega^2 x$, displacement, velocity and acceleration equations and graphs, energy in SHM, and damping, forced oscillations and resonance.

Amplitude, period, frequency, phase | SHM definition $a = -\omega^2 x$ | $x = x_o \sin \omega t$ | $v = \pm\omega\sqrt{(x_o^2 - x^2)}$ | Energy in SHM | Light, critical & heavy damping | Forced oscillations | Resonance

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Paper tag = **where it is examined** DATA & FORMULAE given in the exam C1 A1 B1 how marks are awarded

highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **SHM:** acceleration is proportional to displacement from a fixed point and always directed towards it: $a = -\omega^2 x$.
- **Solutions:** $x = x_o \sin \omega t$ (starting at the centre) or $x_o \cos \omega t$ (starting at an extreme). $\omega = 2\pi f = 2\pi/T$.
- **Speed:** $v = \pm\omega\sqrt{(x_o^2 - x^2)}$, so $v_o = \omega x_o$ at the centre and zero at the extremes. Maximum acceleration $\omega^2 x_o$ is at the extremes.
- **Energy:** total $E = \frac{1}{2}m\omega^2 x_o^2$, constantly swapping between kinetic and potential. Damping removes energy; resonance happens when the driving frequency equals the natural frequency.

BY THE END YOU CAN

- Understand amplitude, period, frequency, angular frequency and phase difference.
- Define SHM by $a = -\omega^2 x$ and use $x = x_o \sin \omega t$, $v = v_o \cos \omega t$ and $v = \pm\omega\sqrt{(x_o^2 - x^2)}$.
- Describe the interchange of kinetic and potential energy, and use $E = \frac{1}{2}m\omega^2 x_o^2$.
- Describe light, critical and heavy damping, forced oscillations and resonance.

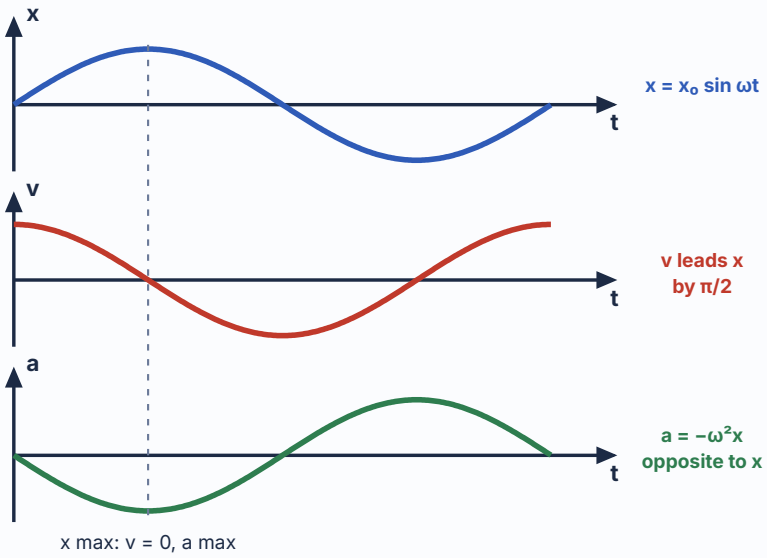
1 Simple harmonic motion

The defining equation

Simple harmonic motion — motion in which the acceleration is proportional to the displacement from a fixed point and is directed towards that point.

$$a = -\omega^2 x \; ; \; x = x_0 \sin \omega t \; ; \; v = v_0 \cos \omega t \; ; \; v = \pm\omega\sqrt{(x_0^2 - x^2)}$$ DATA & FORMULAE

$$\omega = 2\pi f, \; v_o = \omega x_o$$



x, v and a are each a quarter-cycle apart: v leads x by π/2, and a is in antiphase with x.

- **Phase difference:** the fraction of a cycle between two oscillations, as an angle (2π rad = one cycle).
- **The period of SHM doesn't depend on the amplitude** (isochronous) — the basis of pendulum clocks.

WORKED EXAMPLE

A mass on a spring oscillates in SHM at 2.5 Hz with amplitude 0.040 m. Find (a) the maximum speed, (b) the maximum acceleration, (c) the speed at x = 0.020 m.

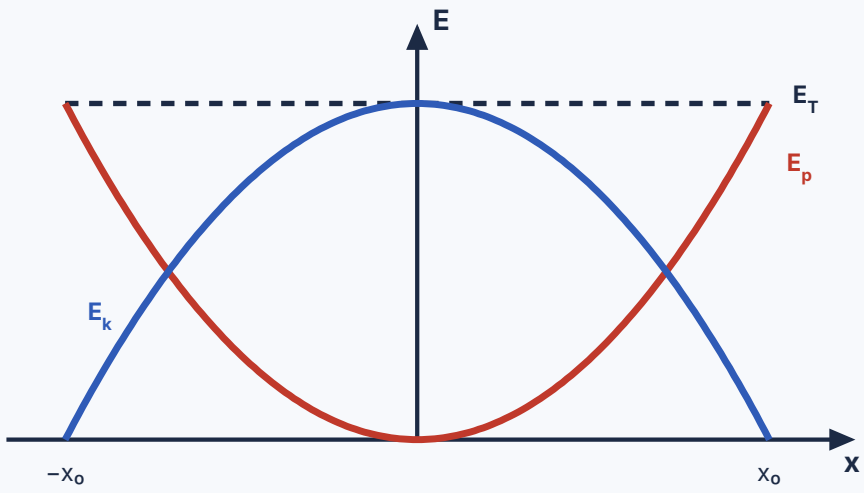
1. $\omega = 2\pi \times 2.5 = 15.7$ rad s^{-1} C1
2. (a) $v_o = \omega x_o =$ **0.63 m s^{-1}** A1
3. (b) $a_{max} = \omega^2 x_o =$ **9.9 m s^{-2}** A1
4. **(c) $v = 15.7\sqrt{(0.040^2 - 0.020^2)} = 0.54$ m s^{-1}** A1

COMMON MISTAKE

Calculator in degree mode for $x = x_o \sin \omega t$. ωt is in radians.

2 Energy and damping

Energy in SHM



$E_p = \frac{1}{2}m\omega^2x^2$, $E_k = \frac{1}{2}m\omega^2(x_o^2 - x^2)$: they add to $E_T = \frac{1}{2}m\omega^2x_o^2$

Kinetic energy is greatest at the centre; potential energy greatest at the extremes; the total is constant.

$$E_{total} = \tfrac{1}{2}m\omega^2x_0^2 \; ; \; E_K = \tfrac{1}{2}m\omega^2(x_0^2 - x^2)$$

undamped

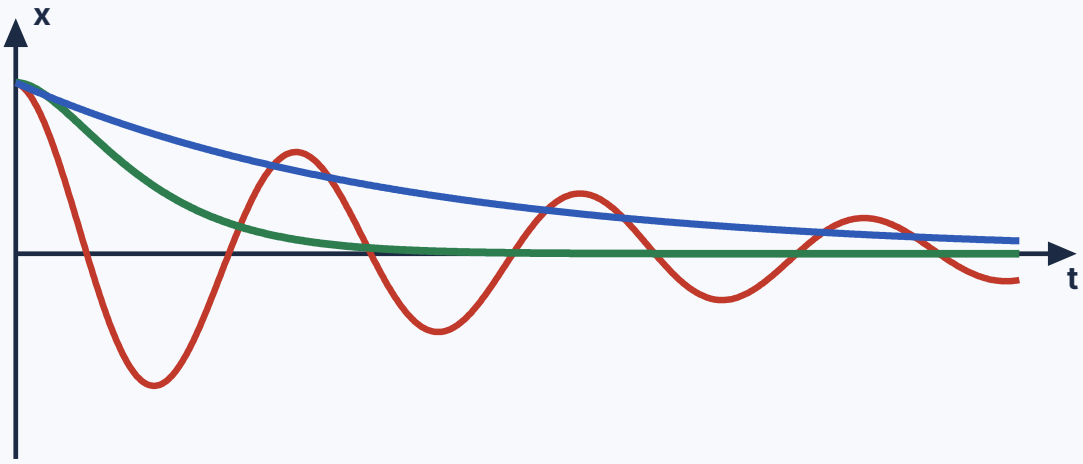
WORKED EXAMPLE

The mass above is 0.30 kg. Find the total energy of the oscillation.

1. $E = \frac{1}{2} \times 0.30 \times 15.7^2 \times 0.040^2$ C1
2. **$E = 0.059$ J** A1

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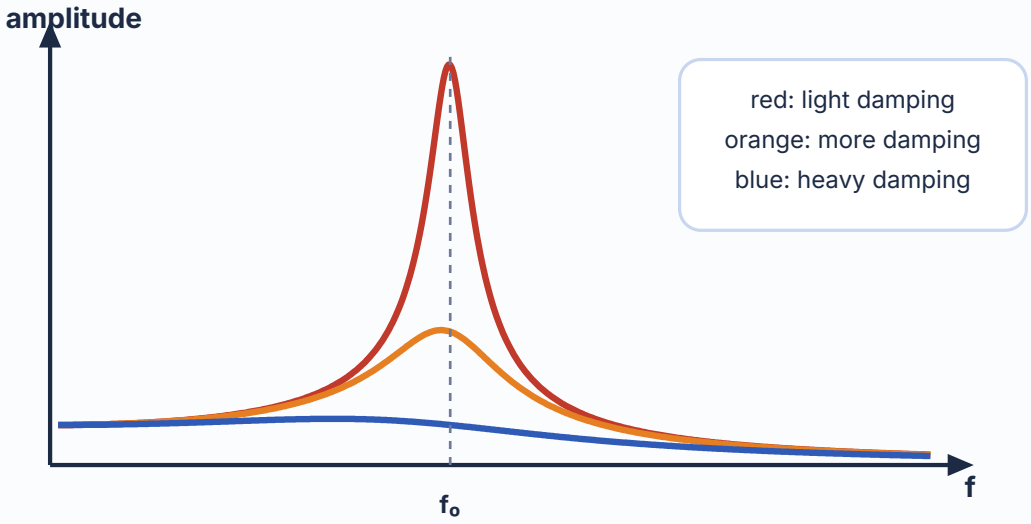
Damping and resonance



Light damping: amplitude decays gradually; critical damping: returns to equilibrium fastest without oscillating.

Damping	What happens
Light	oscillates with gradually decreasing amplitude; the period is almost unchanged
Critical	returns to equilibrium in the shortest time without oscillating (e.g. car suspension)
Heavy	returns to equilibrium slowly, without oscillating

Resonance — the maximum amplitude of a forced oscillation, which occurs when the driving frequency equals the natural frequency of the system.



More damping: lower, broader peak, shifted slightly below f_o.

More damping lowers and broadens the resonance peak and shifts it slightly lower.

- **Useful resonance:** microwave heating of water, tuning a radio, MRI. **Problem resonance:** bridges, car panels vibrating; reduced by damping.

COMMON MISTAKE

Defining SHM as "a ∝ x" without the direction (towards the fixed point), or describing resonance without "driving frequency = natural frequency". Both lost marks in 2024.

LINKING QUESTION

SHM is the projection of circular motion; wave motion is in Waves; and energy ideas are from Work, energy and power.

Test yourself — cover the answers first

QUICK CHECK

1. State the defining equation of SHM.
2. Where is the speed greatest in SHM?
3. What is the phase difference between x and a in SHM?
4. Which damping returns a system to rest fastest without oscillating?
5. What happens to the energy of an SHM oscillator if its amplitude doubles?

EXAM-STYLE

6. A pendulum has period 2.0 s and amplitude 0.10 m. Find its displacement 0.25 s after passing through the centre.
7. For a body in SHM with $\omega = 4.0$ rad s^{-1}, find the acceleration when $x = -0.050$ m.
8. Explain why heavy damping makes the resonance peak lower and broader.

Show answers

1. $a = -\omega^2 x$
2. at the equilibrium position
3. π rad (antiphase)
4. critical damping
5. it quadruples
6. $\omega = \pi$; $x = 0.10 \sin(\pi \times 0.25) = 0.071$ m
7. $a = -16 \times (-0.050) = +0.80$ m s^{-2}
8. more energy is removed each cycle, so the maximum amplitude is smaller and less sharply dependent on frequency

Stuck on a question?

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