IB DP MATHS AA · TOPIC 5 CALCULUS · SL 5.1–5.8 · AHL 5.12–5.15 · ~20 MIN READ

Differentiation

What a derivative really is, every rule you need (power, chain, product, quotient), tangents and normals, stationary points, concavity, optimisation, and the HL extras: first principles, implicit differentiation, related rates and l'Hôpital's rule.

First principles · Chain, product & quotient rules · Tangents & normals · Stationary points & inflection · Optimisation · Implicit differentiation · Related rates · L'Hôpital's rule

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No tag = **SL and HL** · HL ONLY HL students only · FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded · highlight = comes up again and again · Test yourself at the end

IN 30 SECONDS

- The **derivative** f′(x) gives the **gradient of the curve** and the **rate of change** of y with respect to x. It is the limit of chord gradients as the chord shrinks to a point.
- Four rules do almost everything: the **power rule** (rewrite roots and fractions as powers first), the **chain rule** for a function inside a function, the **product rule** and the **quotient rule**. All four are in the formula booklet.
- **f′ = 0** finds stationary points, the **sign of f′′** classifies them, and **f′′ = 0 with a change of sign** finds points of inflection. Optimisation is the same idea applied to a real quantity.
- **HL:** first principles, continuity and differentiability, implicit differentiation, related rates, l'Hôpital's rule and the derivatives of tan, sec, a^x, $\log_a x$ and the inverse trig functions.
- **2029 syllabus:** nothing in this topic changes. These notes work for exams up to November 2028 and from May 2029.

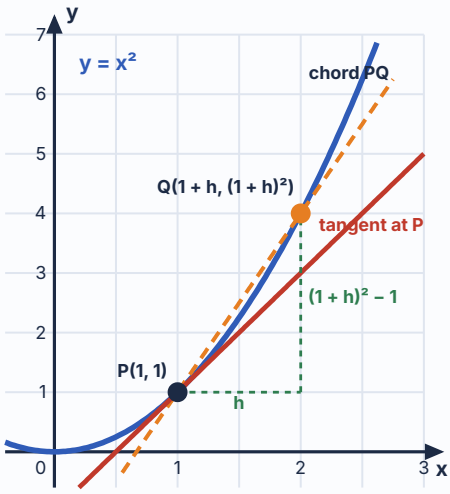
BY THE END YOU CAN

- Explain the derivative as a gradient and as a rate of change, with correct notation.
- Differentiate powers, sin, cos, e^x, ln x and combinations using the chain, product and quotient rules.
- Find equations of tangents and normals, with and without a GDC.
- Find and classify stationary points, and find points of inflection.
- Link the graphs of f, f′ and f′′, and solve optimisation problems with full justification.
- Differentiate from first principles and decide whether a function is differentiable. HL
- Differentiate implicitly, solve related-rates problems and evaluate limits with l'Hôpital's rule. HL

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1 What the derivative means

Gradient of a curve, rate of change



gradient of chord PQ
= ((1 + h)² − 1) ÷ h
= 2 + h → 2 as h → 0
so f′(1) = 2

Slide Q towards P. The chord gradients 2 + h close in on 2, the gradient of the tangent at P. That limit is the derivative.

Derivative — the gradient of the tangent to y = f(x) at a point, which is also the instantaneous rate of change of y with respect to x.

Notation — dy/dx (Leibniz) and f′(x) (Lagrange) mean the same thing. The second derivative is d^2y/dx^2 or f″(x).

- **Units tell the story:** if V is in cm^3 and t in seconds, dV/dt is in **$cm^3\ s^{-1}$**. A question that says "rate of change" is asking for a derivative.
- **Sign matters:** dy/dx > 0 means y increases as x increases; dy/dx < 0 means it decreases.

SYLLABUS NOTE

Differentiation is unchanged for the new syllabus (first exams May 2029). The content removed from AA is financial applications and bivariate statistics, plus Euler's method and proof by counterexample at HL. None of it is in this note.

Differentiation from first principles

HL ONLY

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ FORMULA BOOKLET

the formula is given; the algebra is up to you

- **Method:** (1) write f(x + h) and expand it; (2) subtract f(x), so every term left has a factor h; (3) divide by h; (4) only now let h → 0.

WORKED EXAMPLE

Differentiate $f(x) = x^3$ from first principles.

1. $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$ M1
2. $= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$ A1
3. $= \lim_{h \to 0} (3x^2 + 3xh + h^2)$ A1
4. $= 3x^2$ AG

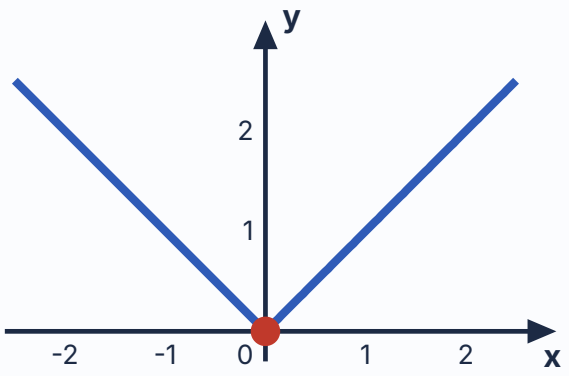
COMMON MISTAKE

Writing "$= 3x^2 + 3xh + h^2 = 3x^2$" without "$\lim_{h \to 0}$" on every line. Examiners want the limit written until the last step, when h actually goes to 0.

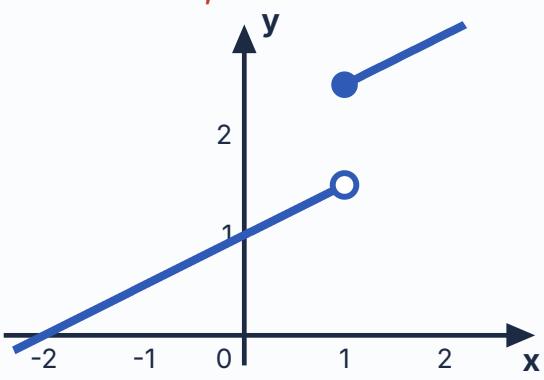
Continuity, differentiability, higher derivatives

HL ONLY

y = |x|: continuous at 0,
but NOT differentiable (corner)



a jump at x = 1:
not continuous, so not differentiable



Differentiable ⇒ continuous. The reverse is false.

A function must be continuous to be differentiable, but continuity is not enough: |x| has no single tangent at its corner.

Continuous at x = a — $\lim_{x \to a} f(x)$ exists and equals f(a): no jump, gap or asymptote there.

Differentiable at x = a — the left-hand and right-hand limits of the chord gradient exist and are equal. Graphically: one smooth tangent.

- **Higher derivatives:** $f^{(n)}(x)$ or $d^n y/dx^n$ means differentiate n times. A pattern in f′, f″, f‴ often leads to a proof by induction (Topic 1).

WORKED EXAMPLE

Find a formula for the nth derivative of $y = e^{2x}$.

1. $dy/dx = 2e^{2x}$, $d^2y/dx^2 = 4e^{2x}$, $d^3y/dx^3 = 8e^{2x}$ M1
2. each derivative multiplies by 2
3. **$d^n y/dx^n = 2^n e^{2x}$** A1

2 Differentiation rules

Powers of x

$$y = x^n \Rightarrow dy/dx = nx^{n-1}$$ FORMULA BOOKLET

n can be negative or a fraction

- **Rewrite first, then differentiate.** $\sqrt{x} = x^{1/2}$, $1/x^3 = x^{-3}$, $5/(2\sqrt{x}) = (5/2)x^{-1/2}$. Split a fraction over a single term: $(x^2 + 1)/x = x + x^{-1}$.
- **Constants:** the derivative of a constant is 0, and a constant multiplier stays: $d/dx\,(7x^4) = 28x^3$.

WORKED EXAMPLE

Differentiate $y = \dfrac{3x^2 - 4}{\sqrt{x}}$.

1. rewrite: $y = 3x^{3/2} - 4x^{-1/2}$ M1
2. $dy/dx = \dfrac{9}{2}x^{1/2} + 2x^{-3/2}$ A1 A1
3. $= \dfrac{9\sqrt{x}}{2} + \dfrac{2}{x\sqrt{x}}$

COMMON MISTAKE

Differentiating the top and bottom of a fraction separately. That is never a rule. Either rewrite as powers or use the quotient rule.

Standard derivatives FORMULA BOOKLET

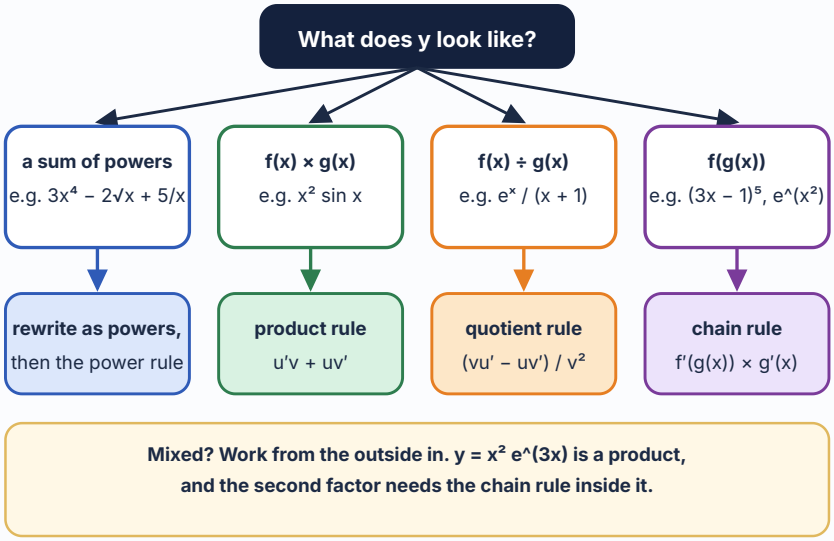
f(x)	f′(x)	note
x^n	nx^{n-1}	any rational n at SL
sin x	cos x	x in radians
cos x	−sin x	note the minus sign
e^x	e^x	its own derivative
ln x	1/x	x > 0

- **Linear inside:** $d/dx\,f(ax + b) = a\,f'(ax + b)$. So $d/dx\,\sin(3x) = 3\cos(3x)$, $d/dx\,e^{5x-1} = 5e^{5x-1}$ and $d/dx\,\ln(2x) = 2/(2x) = 1/x$.

COMMON MISTAKE

Using degrees. Every calculus result for sin and cos assumes **radians**. Set your GDC to radian mode before Paper 2.

Choosing the rule



Mixed? Work from the outside in. $y = x^2 e^{3x}$ is a product, and the second factor needs the chain rule inside it.

Ask what the last operation is if you were evaluating y. That operation decides the rule.

Chain rule

$$y = g(u),\ u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

FORMULA BOOKLET

differentiate the outside, keep the inside, multiply by the derivative of the inside

WORKED EXAMPLE

Differentiate $y = (2x^3 - 5)^4$.

1. let $u = 2x^3 - 5$, so $y = u^4$ M1
2. $\frac{dy}{du} = 4u^3$ and $\frac{du}{dx} = 6x^2$
3. $\frac{dy}{dx} = 4(2x^3 - 5)^3 \times 6x^2$ A1
4. $= 24x^2(2x^3 - 5)^3$

WORKED EXAMPLE

Differentiate $y = e^{\sin x}$ and $y = \ln(3x^2 + 1)$.

1. $e^{\sin x}$: outside e^u, inside $\sin x$
2. $\frac{dy}{dx} = \cos x \cdot e^{\sin x}$ A1
3. $\ln(u)$: derivative is u'/u
4. $\frac{dy}{dx} = \frac{6x}{3x^2 + 1}$ A1

MEMORY TRICK

In shortcut: $\frac{d}{dx}\ln(f(x)) = f'(x)/f(x)$. It turns up again in integration, in reverse.

Product rule

$$y = uv \Rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

FORMULA BOOKLET

two factors that both contain x

WORKED EXAMPLE

Differentiate $y = x^2e^{3x}$.

1. $u = x^2$, $v = e^{3x}$; $u' = 2x$, $v' = 3e^{3x}$ M1
2. $\frac{dy}{dx} = x^2 \cdot 3e^{3x} + e^{3x} \cdot 2x$ A1
3. $= xe^{3x}(3x + 2)$, factorised

EXAM TIP

Factorise your answer. The next part of the question usually asks where $\frac{dy}{dx} = 0$, and a factorised derivative solves itself: $e^{3x} > 0$, so $x = 0$ or $x = -2/3$.

Quotient rule

$$y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

FORMULA BOOKLET

the order on the top matters

WORKED EXAMPLE

Find the coordinates of the stationary point of $y = \frac{\ln x}{x}$, $x > 0$.

1. $u = \ln x$, $v = x$; $u' = 1/x$, $v' = 1$ M1
2. $\frac{dy}{dx} = \frac{x \cdot (1/x) - \ln x \cdot 1}{x^2} = \frac{1 - \ln x}{x^2}$ A1
3. $\frac{dy}{dx} = 0 \Rightarrow \ln x = 1 \Rightarrow x = e$ M1
4. $y = 1/e$, so the point is $(e, 1/e)$ A1

MEMORY TRICK

"Bottom times derivative of top, minus top times derivative of bottom, all over bottom squared." The bottom goes first.

More standard derivatives

HL ONLY

f(x)	f′(x)
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$
$\cot x$	$-\operatorname{cosec}^2 x$
a^x	$a^x \ln a$
$\log_a x$	$1/(x \ln a)$
$\arcsin x$	$1/\sqrt{(1 - x^2)}$
$\arccos x$	$-1/\sqrt{(1 - x^2)}$
$\arctan x$	$1/(1 + x^2)$

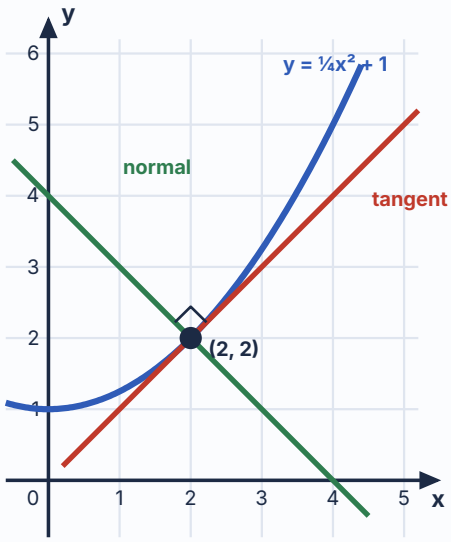
- All of these are in the formula booklet FORMULA BOOKLET, but you can be asked to **prove** one. $\tan x = \sin x/\cos x$ by the quotient rule gives $(\cos^2 x + \sin^2 x)/\cos^2 x = 1/\cos^2 x = \sec^2 x$.

WORKED EXAMPLE

Differentiate $y = \arctan(2x)$ and $y = 3^{x^{\wedge}\{2\}}$.

1. $\arctan(u)$ with $u = 2x$: $\dfrac{dy}{dx} = \dfrac{1}{1 + (2x)^2} \times 2 = \dfrac{2}{1 + 4x^2}$ A1
2. a^u with $u = x^2$: $\dfrac{dy}{dx} = 3^{x^{\wedge}\{2\}} \ln 3 \times 2x = 2x \ln 3 \cdot 3^{x^{\wedge}\{2\}}$ A1

3 Tangents and normals



f′(x) = x/2, so at x = 2:
tangent gradient m = 1
normal gradient = −1/m = −1
tangent: y = x
normal: y = −x + 4

The normal is perpendicular to the tangent, so the two gradients multiply to −1.

- **Method:** (1) find the point (a, f(a)); (2) find the gradient m = f′(a); (3) tangent: y − f(a) = m(x − a); (4) normal: gradient −1/m, same point.

WORKED EXAMPLE

Find the equation of the normal to $y = x^3 - 2x$ at $x = 1$.

1. point: $y = 1 - 2 = -1$, so $(1, -1)$ A1
2. $dy/dx = 3x^2 - 2$, at $x = 1$ it is 1 M1 A1
3. normal gradient $= -1$ M1
4. **$y + 1 = -(x - 1)$, so $y = -x$** A1

GDC (CALCULATOR) METHOD

Paper 2: find f′(a) with the numerical derivative (nDeriv or dy/dx on the graph screen of a TI-84; the d/dx template on TI-Nspire and Casio). Write "f′(2) = 3.41..." in your working, then the equation. A bare final answer can lose the method mark.

EXAM TIP

Give the equation in the form asked. "In the form ax + by + d = 0 where a, b, d ∈ ℤ" means clear the fractions: y = −¾x + 6 becomes 3x + 4y − 24 = 0.

4 Stationary points and the shape of a graph

Increasing and decreasing

- **f′(x) > 0** on an interval: f is increasing there. **f′(x) < 0**: decreasing. Answer with an interval, such as 1 < x < 3.

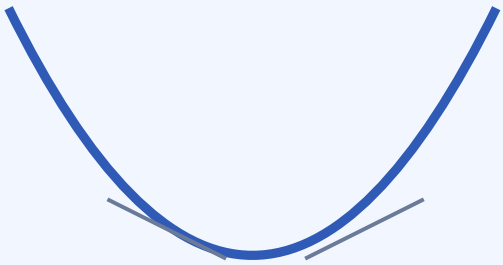
WORKED EXAMPLE

Find where $f(x) = x^3 - 6x^2 + 9x + 1$ is decreasing.

1. $f'(x) = 3x^2 - 12x + 9 = 3(x - 1)(x - 3)$ M1
2. $f'(x) < 0$ between the roots R1
3. **$1 < x < 3$** A1

Concavity and the second derivative

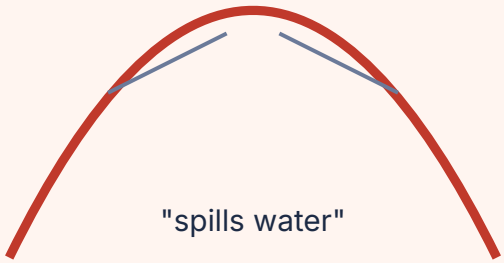
concave up: f″(x) > 0



"holds water"

gradient increasing

concave down: f″(x) < 0



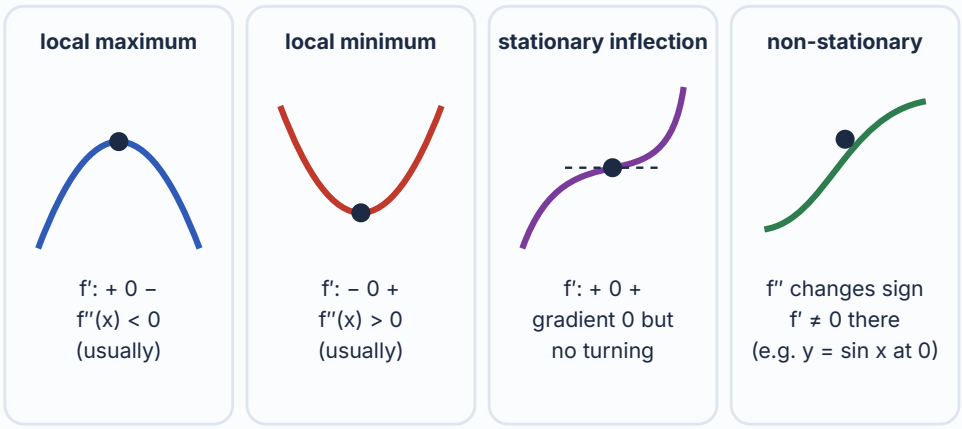
"spills water"

gradient decreasing

f″ is the rate of change of the gradient. Positive f″ means the gradient is increasing, so the curve bends upwards.

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Classifying stationary points



Stationary point: f′(x) = 0. Point of inflection: f″(x) = 0 AND f″ changes sign.

f″ = 0 alone is not enough: for $y = x^4$, f″(0) = 0 but (0, 0) is a minimum.

Four shapes to recognise. The sign of f′ either side settles every case, even when f″ = 0.

- **Second derivative test:** at a stationary point, f″(a) < 0 ⇒ local maximum; f″(a) > 0 ⇒ local minimum; f″(a) = 0 ⇒ **the test fails**, so check the sign of f′ either side.

WORKED EXAMPLE

Find and classify the stationary points of $f(x) = x^4 - 4x^3$, and find any points of inflection.

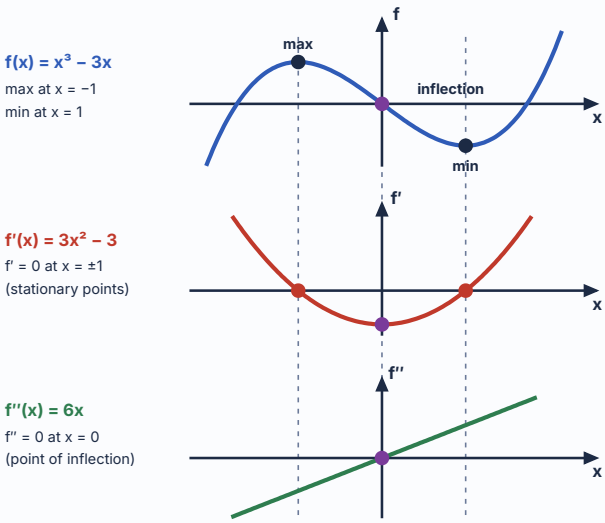
1. $f'(x) = 4x^3 - 12x^2 = 4x^2(x - 3) = 0 \Rightarrow x = 0$ or $x = 3$ M1 A1
2. $f''(x) = 12x^2 - 24x = 12x(x - 2)$ A1
3. $x = 3$: $f''(3) = 36 > 0$, so **(3, −27) is a minimum** R1
4. $x = 0$: $f''(0) = 0$, so test the sign of f′: negative just left and just right of 0 M1
5. so **(0, 0) is a stationary point of inflection** A1
6. **f″ = 0 at x = 2 and changes sign there, so (2, −16) is a non-stationary point of inflection** A1

COMMON MISTAKE

Saying "f″(0) = 0, so it is a point of inflection". f″ = 0 is necessary but not sufficient: $y = x^4$ has f″(0) = 0 at a minimum. Always check that f″ changes sign.

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Graphs of f, f' and f"



Read down a column: where f has a turning point, f′ crosses 0; where f changes the way it bends, f″ crosses 0.

The same three x-values line up in all three graphs.

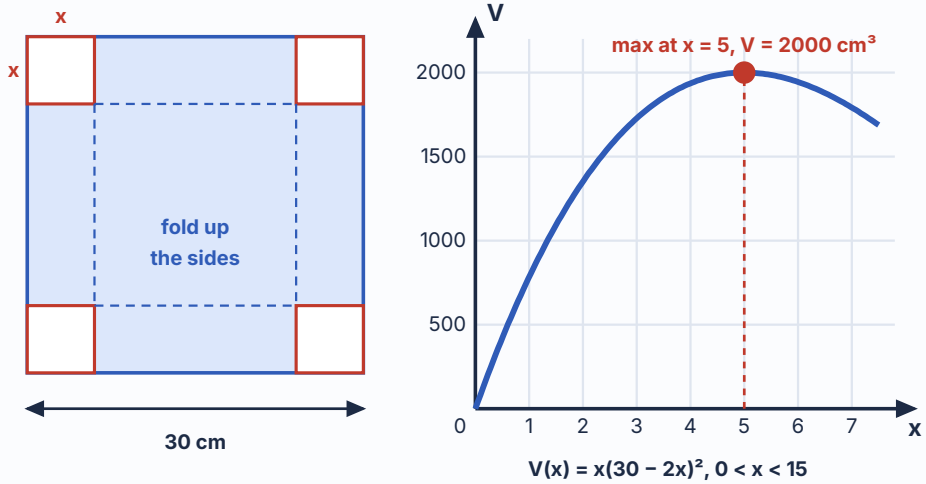
On the graph of f	On the graph of f′
local maximum	crosses the x-axis from + to −
local minimum	crosses the x-axis from − to +
increasing	above the x-axis
point of inflection	a turning point
stationary inflection	touches the x-axis without crossing

EXAM TIP

Paper 1 loves "here is the graph of f′; sketch f" (or the reverse). Mark the x-intercepts of f′ first: they are the x-coordinates of the turning points of f.

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5 Optimisation



Squares of side x are cut from the corners of a 30 cm sheet and the sides folded up. Which x gives the biggest box?

- **Method:** (1) name the variable and write the quantity to optimise; (2) use the constraint to get **one variable**; (3) state the domain; (4) solve dQ/dx = 0; (5) **justify** max or min; (6) answer in context, with units.

WORKED EXAMPLE

Find the maximum volume of the open box.

1. $V = x(30 - 2x)^2 = 4x^3 - 120x^2 + 900x$, for $0 < x < 15$ M1 A1
2. $dV/dx = 12x^2 - 240x + 900 = 12(x - 5)(x - 15) = 0$ M1
3. $x = 15$ gives no box, so **$x = 5$** R1
4. $d^2V/dx^2 = 24x - 240 = -120 < 0$ at $x = 5$, so a maximum R1
5. **$V = 5 \times 20^2 = 2000\ cm^3$** A1

WORKED EXAMPLE

A closed cylinder holds 500 cm^3. Find the radius that uses the least metal.

1. $\pi r^2 h = 500$, so $h = 500/(\pi r^2)$ M1
2. $A = 2\pi r^2 + 2\pi rh = 2\pi r^2 + 1000/r$ A1
3. $dA/dr = 4\pi r - 1000/r^2 = 0 \Rightarrow r^3 = 250/\pi$ M1
4. **$r = 4.30$ cm** (3 s.f.), and then $h = 8.60$ cm $= 2r$ A1
5. **$d^2A/dr^2 = 4\pi + 2000/r^3 > 0$, so a minimum** R1

COMMON MISTAKE

Stopping at "x = 5". The question asked for the maximum **volume**, and you must justify that it is a maximum. With a closed interval, also compare the endpoints: the largest value can sit at an end.

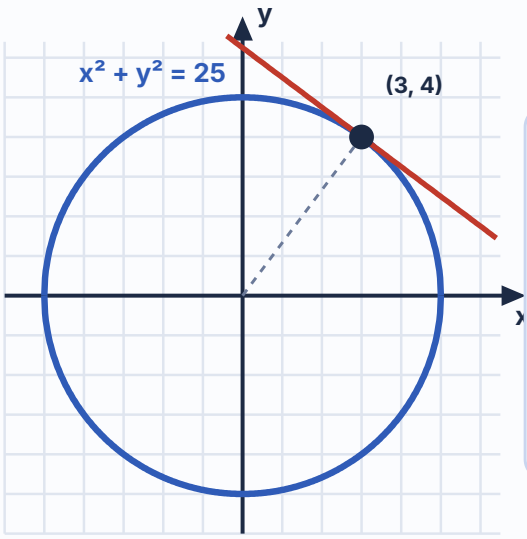
GDC (CALCULATOR) METHOD

On Paper 2, graph the function and use the maximum or minimum tool to check your answer. Still write the derivative equation you are solving.

6 Implicit differentiation and related rates HL

Implicit differentiation

HL ONLY



differentiate every term in x:
2x + 2y dy/dx = 0
dy/dx = −x/y
at (3, 4): dy/dx = −3/4
tangent: y − 4 = −¾(x − 3)
check: the radius has
gradient 4/3, and
4/3 × (−3/4) = −1 ✓

You do not need to make y the subject. Differentiate both sides with respect to x.

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx}$$

it is the chain rule, because y depends on x

- **Products:** $\frac{d}{dx}(xy) = x\frac{dy}{dx} + y$. **Then** collect the dy/dx terms on one side and factorise.

WORKED EXAMPLE

Find the gradient of $x^2 + xy + y^2 = 7$ at (1, 2).

1. $2x + (x\frac{dy}{dx} + y) + 2y\frac{dy}{dx} = 0$ M1 A1
2. $\frac{dy}{dx}(x + 2y) = -(2x + y)$ M1
3. $\frac{dy}{dx} = -\frac{2x + y}{x + 2y} = -\frac{4}{5}$ at (1, 2) A1

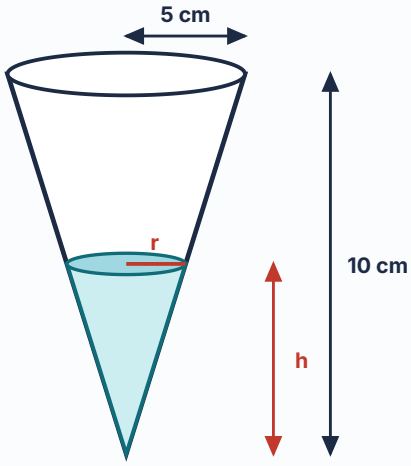
WORKED EXAMPLE

Show that $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{(1 - x^2)}}$.

1. $y = \arcsin x$, so $\sin y = x$ M1
2. $\cos y \frac{dy}{dx} = 1$, so $\frac{dy}{dx} = \frac{1}{\cos y}$ A1
3. $\cos y = \sqrt{(1 - \sin^2 y)} = \sqrt{(1 - x^2)}$, positive because $-\pi/2 \le y \le \pi/2$ R1
4. $\frac{dy}{dx} = \frac{1}{\sqrt{(1 - x^2)}}$ AG

Related rates of change

HL ONLY



water pours in at 3 $cm^3\ s^{-1}$

similar triangles: $r/h = 5/10$

so $r = h/2$

$V = \frac{1}{3}\pi r^2 h = \pi h^3/12$

$dV/dh = \pi h^2/4$

$dV/dt = dV/dh \times dh/dt$

$3 = (\pi \times 4^2/4) \times dh/dt$ at $h = 4$

$dh/dt = 3/(4\pi) \approx 0.239\ cm\ s^{-1}$

The volume grows at a steady rate, but the water level rises more slowly as the cone gets wider.

- **Method:** (1) write an equation linking the variables; (2) differentiate it **with respect to t**; (3) substitute the numbers **last**.

WORKED EXAMPLE

A 5 m ladder leans on a wall. Its foot slides away at 0.4 m s^{-1}. How fast is the top sliding down when the foot is 3 m from the wall?

1. $x^2 + y^2 = 25$ M1
2. $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$ M1 A1
3. when $x = 3$, $y = 4$: $2(3)(0.4) + 2(4)\frac{dy}{dt} = 0$ M1
4. $\frac{dy}{dt} = -0.3$, **so the top slides down at 0.3 m s^{-1}** A1

COMMON MISTAKE

Substituting $x = 3$ before differentiating. Then x is a constant, its derivative is 0, and the rate disappears from the equation.

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7 Limits and l'Hôpital's rule HL

$$\text{if } \lim \frac{f(x)}{g(x)} \text{ gives } \frac{0}{0} \text{ or } \frac{\infty}{\infty}, \text{ then } \lim \frac{f(x)}{g(x)} = \lim \frac{f'(x)}{g'(x)}$$ LEARN IT

repeat while the limit is still indeterminate

- Differentiate the top and the bottom **separately**. This is not the quotient rule.

WORKED EXAMPLE

Evaluate $\lim_{x \to 0} \frac{1 - \cos x}{x^2}$.

1. substituting gives 0/0 R1
2. $= \lim_{x \to 0} \frac{\sin x}{2x}$, still 0/0 M1
3. $= \lim_{x \to 0} \frac{\cos x}{2}$ M1
4. $= \frac{1}{2}$ A1

WORKED EXAMPLE

Evaluate $\lim_{x \to \infty} \frac{x^2}{e^x}$.

1. ∞/∞, so l'Hôpital: $\lim \frac{2x}{e^x}$ M1
2. still ∞/∞: $\lim \frac{2}{e^x}$ M1
3. **= 0, so e^x grows faster than any power** A1

COMMON MISTAKE

Using l'Hôpital when the limit is not 0/0 or ∞/∞. $\lim_{x \to 0} (x + 1)/(x + 2)$ is simply 1/2 by substitution; differentiating would give 1, which is wrong.

- Maclaurin series give another route to the same limits. They have their own HL note.

Riyaz Sir

8 Exam technique

Paper 1, Paper 2 and command terms

- **Paper 1 (no calculator):** exact answers such as 1/e, ln 3 or π/6. Factorise derivatives so f′ = 0 solves by hand.
- **Paper 2 (calculator):** use the GDC to evaluate f′(a), find roots of f′(x) = 0 and find maxima, but write the equation you solved. "Correct answer, no working" earns little if the answer is wrong.
- **"Show that":** every step on the page, ending exactly at the given result. Marked AG: no marks for the final line itself.
- **"Hence":** use the result you just found. Starting again from scratch can score zero.
- **"Justify":** a reason earns the R1 mark, for example "f″(5) = −120 < 0, so it is a maximum" or "f′ changes from positive to negative".

LINKING QUESTION

Velocity is the derivative of displacement and acceleration is the derivative of velocity. The same calculus drives IB Physics A.1 Kinematics, and the integration note runs it in reverse.

IDEA FOR YOUR IA / EXPLORATION

Optimisation makes a strong exploration: is a 330 ml drinks can the shape that uses the least aluminium? Measure a real can, model its surface area, compare with the optimum h = 2r, and explain the difference (rims, strength, stacking).

Test yourself — cover the answers first

QUICK CHECK

1. Differentiate $y = 4x^3 - \frac{2}{x}$.
2. Find f′(4) for $f(x) = \sqrt{x}$.
3. Differentiate sin(3x) and e^{-2x}.
4. Differentiate $y = xe^x$.
5. Differentiate $y = \ln(x^2 + 1)$.
6. The tangent at P has gradient −2/3. What is the gradient of the normal?
7. If f″(a) = 0, must x = a be a point of inflection?
8. HL Differentiate tan(2x).
9. HL Find the gradient of y = arctan x at x = 1.

EXAM-STYLE

10. A farmer fences three sides of a rectangle with 120 m of fence; a river forms the fourth side. Find the maximum area.
11. Let $f(x) = xe^{-x}$. (a) Find f′(x). (b) Find the coordinates of the maximum and justify it. (c) Find the point of inflection.
12. HL Find the equation of the tangent to $x^3 + y^3 = 9xy$ at (2, 4).
13. HL Evaluate $\lim_{x \to 0} \frac{e^x - 1 - x}{x^2}$.
14. HL Air is pumped into a spherical balloon at 100 $cm^3\ s^{-1}$. How fast is the radius increasing when r = 5 cm?

Show answers

1. $12x^2 + \frac{2}{x^2}$
2. $f'(x) = \frac{1}{2\sqrt{x}}$, so $f'(4) = \frac{1}{4}$
3. $3\cos(3x)$ and $-2e^{-2x}$
4. $e^x(x + 1)$, by the product rule
5. $\frac{2x}{x^2 + 1}$
6. $\frac{3}{2}$
7. No. f'' must also change sign ($y = x^4$ has a minimum at 0).
8. $2\sec^2(2x)$
9. $\frac{1}{1+1} = \frac{1}{2}$
10. two sides x and one side $120 - 2x$, so $A = x(120 - 2x)$
    $A'(x) = 120 - 4x = 0 \Rightarrow x = 30$
    $A'' = -4 < 0$, so a maximum
    maximum area $= 30 \times 60 =$ **1800 m^2**
11. (a) $f'(x) = e^{-x}(1 - x)$
    (b) $f' = 0 \Rightarrow x = 1$; $f''(x) = e^{-x}(x - 2)$, $f''(1) = -1/e < 0$, so a maximum at **(1, 1/e)**
    (c) $f'' = 0$ at $x = 2$ and changes sign there: **(2, 2/e^2)**
12. $3x^2 + 3y^2\,y' = 9y + 9xy'$
    $y' = (9y - 3x^2)/(3y^2 - 9x) = 24/30 = 4/5$
    $y - 4 = \frac{4}{5}(x - 2)$, that is $4x - 5y + 12 = 0$
13. 0/0: lim $(e^x - 1)/(2x)$, still 0/0
    lim $e^x/2 =$ **$\frac{1}{2}$**
14. $V = \frac{4}{3}\pi r^3$, so $dV/dt = 4\pi r^2\,dr/dt$
    $100 = 4\pi(25)\,dr/dt$
    $dr/dt = \frac{1}{\pi} \approx$ **0.318 cm s^{-1}**

Stuck on a question?

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Want regular help? Send this to your parent →

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Ask Riyaz Sir