IB DP MATHS AA · TOPIC 2 FUNCTIONS · SL 2.1–2.5 · AHL 2.14 · ~6 MIN READ

Functions, inverses and composites

Straight lines and their equations, what a function is, domain and range, key features of graphs, composite functions, inverse functions and their graphs, and (HL) odd and even functions, self-inverse functions and restricting domains.

Straight lines | Domain & range | Key features of graphs | Composite functions | Inverse functions | Odd & even functions (HL) | Self-inverse (HL)

RS by Riyaz Sir
mentorriyaz.in

No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- A **function** maps every input in its **domain** to exactly one output; the set of outputs is the **range**.
- A **composite** $(f \circ g)(x) = f(g(x))$ applies g first, then f. Order matters.
- An **inverse** f^{-1} undoes f. It exists only if f is one-to-one; its graph is the reflection of f in **y = x**, and domain and range swap.
- **Lines:** gradient $m = \Delta y/\Delta x$; parallel lines have equal gradients, perpendicular lines have $m_1 m_2 = -1$.
- **HL:** even functions $(f(-x) = f(x))$, odd functions $(f(-x) = -f(x))$, self-inverse functions $(f = f^{-1})$ and restricting domains so an inverse exists.

BY THE END YOU CAN

- Find gradients and equations of lines, including parallel and perpendicular lines.
- Find the domain and range of a function and identify key features of its graph.
- Form and evaluate composite functions.
- Find inverse functions algebraically and sketch them by reflection in y = x.
- Identify odd, even and self-inverse functions and restrict a domain to make an inverse exist. HL

1 Straight lines

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ FORMULA BOOKLET

gradient between two points

$$y = mx + c,\ ax + by + d = 0,\ y - y_1 = m(x - x_1)$$ FORMULA BOOKLET

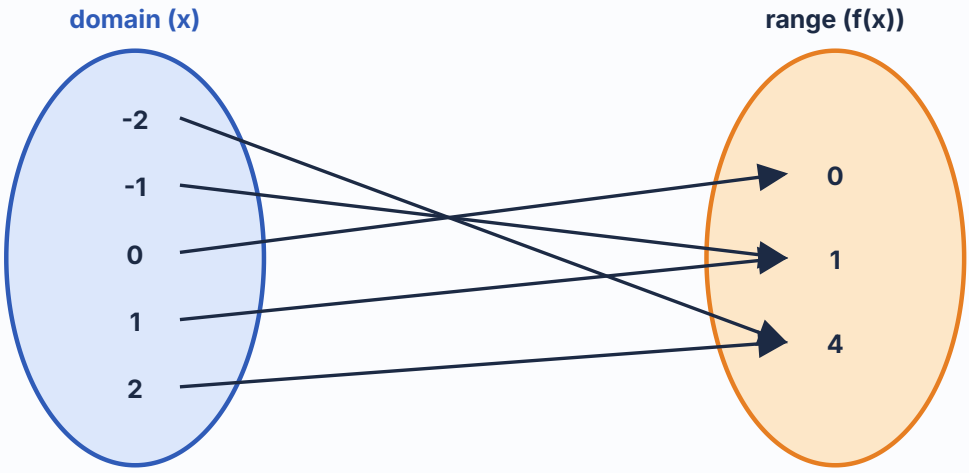
three forms of a line

WORKED EXAMPLE

Find the line through (2, 3) perpendicular to y = 2x + 1, in the form ax + by + d = 0.

1. perpendicular gradient = −½ A1
2. y − 3 = −½(x − 2) M1
3. **x + 2y − 8 = 0** A1

2 Functions, domain and range



$f(x) = x^2$: every input has exactly one output (a function), but outputs repeat, so no inverse.

One output per input: a function. Two inputs to one output is fine; one input to two outputs is not.

- **Domain problems:** you cannot divide by zero, take the square root of a negative, or take the log of zero or a negative.

f(x)	Domain	Range
$\sqrt{(x - 2)}$	$x \geq 2$	$f(x) \geq 0$
$\frac{1}{x - 3}$	$x \neq 3$	$f(x) \neq 0$
$\ln(x + 1)$	$x > -1$	$f(x) \in \mathbb{R}$
$x^2 + 4$	$x \in \mathbb{R}$	$f(x) \geq 4$

- **Key features** (SL 2.3–2.4): intercepts, maximum and minimum points, symmetry, vertical and horizontal asymptotes. Sketch them, label them with coordinates or equations.

GDC (CALCULATOR) METHOD

Paper 2: graph, then use zero, maximum, minimum and intersect. Set the window to show every key feature, and copy the coordinates to 3 s.f.

Riyaz Sir

3 Composite functions

$$(f \circ g)(x) = f(g(x))$$

do g first, then f

WORKED EXAMPLE

$f(x) = 2x + 1$ and $g(x) = x^2$. Find $(f \circ g)(x)$, $(g \circ f)(x)$ and $(f \circ g)(3)$.

1. $(f \circ g)(x) = f(x^2) = \mathbf{2x^2 + 1}$ A1
2. $(g \circ f)(x) = (2x + 1)^2$ A1
3. **$(f \circ g)(3) = 2(9) + 1 = 19$** A1

COMMON MISTAKE

Thinking $f \circ g$ means "f first". The function written closest to x acts first: in $f(g(x))$, g acts on x.

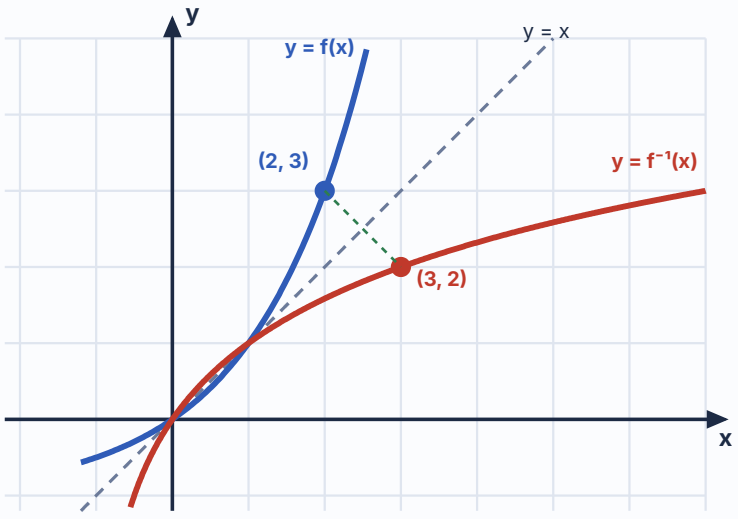
4 Inverse functions

- **Method:** write $y = f(x)$, swap x and y (or rearrange for x), make y the subject, write $f^{-1}(x) = \ldots$

WORKED EXAMPLE

Find $f^{-1}(x)$ for $f(x) = \dfrac{2x + 3}{x - 1}$, $x \neq 1$.

1. $y(x - 1) = 2x + 3 \Rightarrow xy - 2x = y + 3$ M1
2. $x = \dfrac{y + 3}{y - 2}$ A1
3. **$f^{-1}(x) = \dfrac{x + 3}{x - 2}$, $x \neq 2$** A1



(a, b) on f ⇔ (b, a) on f^{-1}. Domain of f^{-1} = range of f.

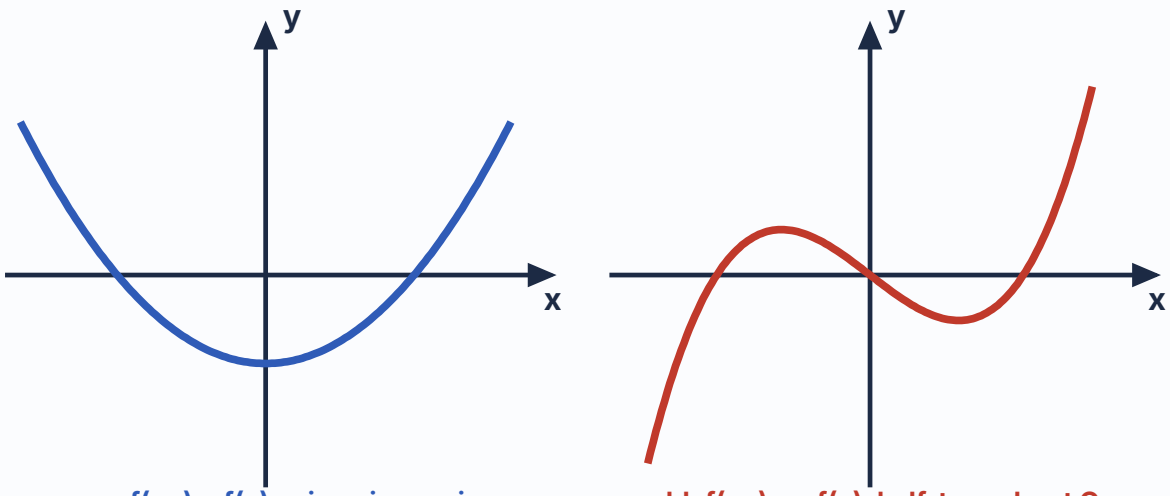
Reflect every point (a, b) to (b, a): that is reflection in y = x.

- **$f(f^{-1}(x)) = f^{-1}(f(x)) = x$.** The domain of f^{-1} is the range of f, and its range is the domain of f.

EXAM TIP

"Solve $f(x) = f^{-1}(x)$" is quickest by solving $f(x) = x$, when the graphs meet on the line $y = x$ (true for increasing functions).

5 Odd, even and self-inverse functions HL



even: f(−x) = f(x), mirror in y-axis **odd: f(−x) = −f(x), half-turn about O**

Even functions have mirror symmetry in the y-axis; odd functions have half-turn symmetry about the origin.

WORKED EXAMPLE

Is $f(x) = x^3 - 4x$ odd, even or neither?

1. $f(-x) = -x^3 + 4x = -(x^3 - 4x) = -f(x)$ M1
2. **odd** A1

- **Self-inverse:** $f(f(x)) = x$, so the graph is symmetric in $y = x$. Examples: $f(x) = 1/x$, $f(x) = -x + c$, $f(x) = \frac{x+3}{x-1}$.
- **Restricting the domain:** $f(x) = x^2$ has no inverse on $\mathbb{R}$, but on $x \geq 0$ it is one-to-one and $f^{-1}(x) = \sqrt{x}$.

WORKED EXAMPLE

$f(x) = (x-2)^2 + 1$. Find the largest domain of the form $x \geq a$ for which f^{-1} exists, and find f^{-1}.

1. the vertex is at $x = 2$, so **$x \geq 2$** A1
2. $y - 1 = (x-2)^2 \Rightarrow x = 2 + \sqrt{(y-1)}$ (positive root because $x \geq 2$) M1
3. **$f^{-1}(x) = 2 + \sqrt{(x-1)}$, $x \geq 1$** A1

LINKING QUESTION

Inverse functions explain why ln undoes e^x (Exponents and logarithms) and why arcsin needs a restricted range (Trig identities). Composite functions are where the chain rule comes from (Differentiation).

IDEA FOR YOUR IA / EXPLORATION

Model a real process as a composite (currency conversion then tax, temperature conversions) and explore when the inverse makes sense in context.

Test yourself — cover the answers first

QUICK CHECK

1. Find the gradient of the line through (1, 5) and (4, −1).
2. Give the domain of $f(x) = 1/(x + 4)$.
3. If $f(x) = x + 3$ and $g(x) = 2x$, find $g(f(1))$.
4. Find $f^{-1}(x)$ for $f(x) = 5x - 2$.
5. Which line is f^{-1} a reflection in?
6. HL Is cos x odd or even?

EXAM-STYLE

7. $f(x) = 3x - 1$ and $g(x) = \frac{x + 1}{3}$. Show that $g = f^{-1}$ and find $(f \circ g)(x)$.
8. HL $f(x) = e^{2x} - 1$. Find $f^{-1}(x)$ and state its domain.

Show answers

1. −2
2. $x \neq -4$
3. 8
4. $(x + 2)/5$
5. $y = x$
6. even
7. $f(g(x)) = 3(\frac{x + 1}{3}) - 1 = x$
   and $g(f(x)) = x$, so g is the inverse
   $(f \circ g)(x) = \mathbf{x}$
8. $y + 1 = e^{2x} \Rightarrow x = ½ \ln(y + 1)$
   $f^{-1}(x) = ½ \ln(x + 1)$
   domain **$x > -1$** (the range of f)

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

Share with a friend

Ask Riyaz Sir