IB DP MATHS AA · TOPIC 2 FUNCTIONS · SL 2.6, 2.7, 2.10 · ~6 MIN READ

Quadratics and the discriminant

The three forms of a quadratic, completing the square, the vertex and axis of symmetry, solving by factorising and the formula, the discriminant and the number of roots, quadratic inequalities and hidden quadratics.

Standard, factorised & vertex form | Completing the square | Axis of symmetry | Quadratic formula | Discriminant | Inequalities | Hidden quadratics

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

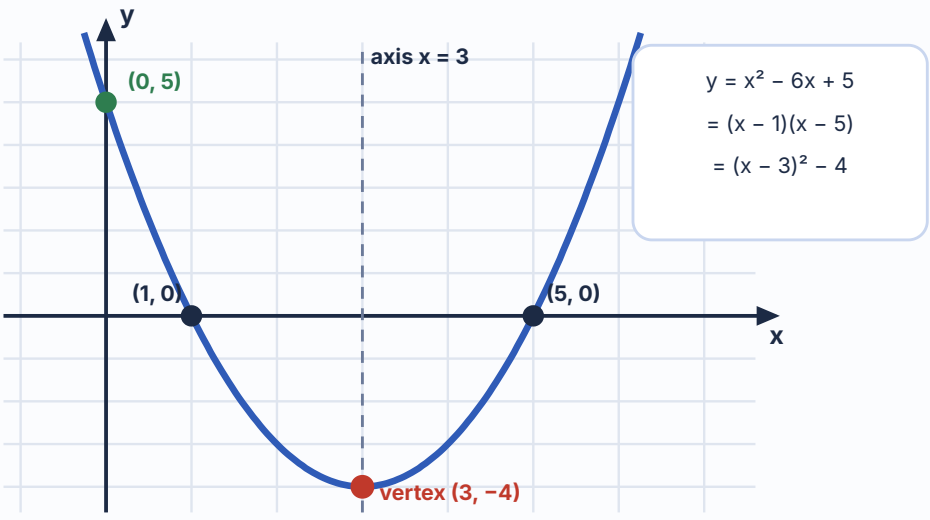
IN 30 SECONDS

- A quadratic $y = ax^2 + bx + c$ has three useful forms: **standard** (y-intercept c), **factorised** $a(x - p)(x - q)$ (roots p and q) and **vertex** $a(x - h)^2 + k$ (vertex (h, k)).
- The **axis of symmetry** is $x = -b/2a$. If $a > 0$ the graph is U-shaped (minimum); if $a < 0$ it is ∩-shaped (maximum).
- The **discriminant $\Delta = b^2 - 4ac$** tells you how many real roots: two ($\Delta > 0$), one repeated ($\Delta = 0$) or none ($\Delta < 0$).
- **Hidden quadratics** such as $x^4 - 5x^2 + 4 = 0$ or $2^{2x} - 6(2^x) + 8 = 0$ become ordinary quadratics with a substitution.
- This note is the same at SL and HL, and unchanged for 2029.

BY THE END YOU CAN

- Convert between standard, factorised and vertex forms, including completing the square.
- Find the vertex, axis of symmetry and intercepts, and sketch the graph.
- Solve quadratic equations by factorising, completing the square and the formula.
- Use the discriminant to decide the number of roots, including in problems with a parameter.
- Solve quadratic inequalities and equations that reduce to quadratics.

1 Forms of a quadratic



One quadratic, three forms. Each form shows a different feature straight away.

$$x = -\frac{b}{2a}$$ FORMULA BOOKLET

axis of symmetry

WORKED EXAMPLE

Write $y = x^2 - 6x + 5$ in vertex form and state the vertex.

1. $x^2 - 6x + 5 = (x - 3)^2 - 9 + 5$ M1
2. $= \mathbf{(x - 3)^2 - 4}$ A1
3. **vertex (3, −4), a minimum** A1

WORKED EXAMPLE

Write $y = 2x^2 + 12x + 7$ in the form $a(x - h)^2 + k$.

1. $= 2(x^2 + 6x) + 7 = 2[(x + 3)^2 - 9] + 7$ M1
2. $= \mathbf{2(x + 3)^2 - 11}$ A1

COMMON MISTAKE

Forgetting to multiply the "− 9" by the 2 outside the bracket. Factor out a first, complete the square inside, then multiply back.

2 Solving quadratic equations

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$ FORMULA BOOKLET

the quadratic formula

WORKED EXAMPLE

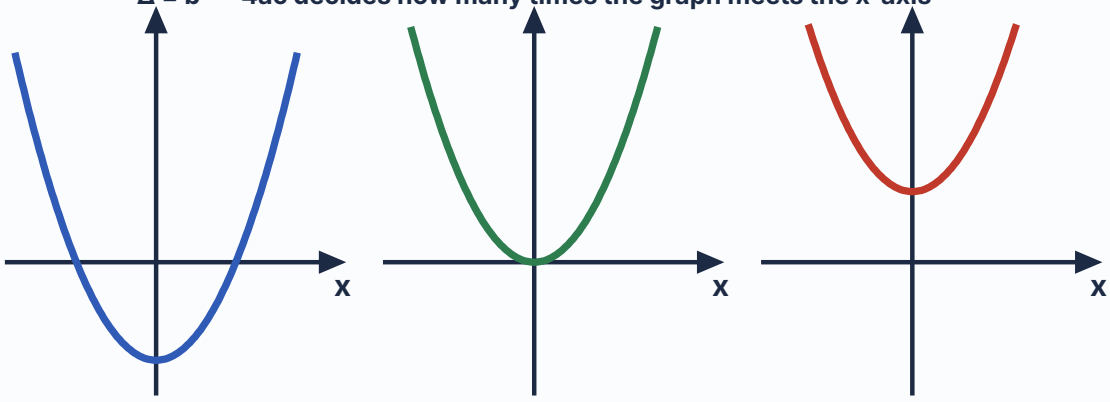
Solve $3x^2 - 5x - 1 = 0$, giving exact answers.

1. $x = \frac{5 \pm \sqrt{(25 + 12)}}{6}$ M1
2. $\mathbf{x = \frac{5 \pm \sqrt{37}}{6}}$ A1

- **Which method?** Factorise if it is obvious; otherwise the formula. Completing the square gives exact roots AND the vertex at the same time.

3 The discriminant

$\Delta = b^2 - 4ac$ decides how many times the graph meets the x-axis



$\Delta > 0$
two real roots

$\Delta = 0$
one repeated root

$\Delta < 0$
no real roots

Shift the same parabola up and the number of real roots drops from two to one to none.

$$\Delta = b^2 - 4ac$$ FORMULA BOOKLET

$\Delta > 0$ two real roots; $\Delta = 0$ one repeated root; $\Delta < 0$ no real roots

WORKED EXAMPLE

Find k so that $x^2 + kx + 9 = 0$ has equal roots.

1. $\Delta = k^2 - 36 = 0$ M1
2. **$k = \pm 6$** A1

WORKED EXAMPLE

Find m so that the line $y = mx$ is tangent to $y = x^2 + 3$.

1. $x^2 - mx + 3 = 0$ must have one repeated root M1
2. $\Delta = m^2 - 12 = 0$ A1
3. **$m = \pm 2\sqrt{3}$** A1

WORKED EXAMPLE

For which k does $kx^2 + 4x + k = 0$ have two distinct real roots?

1. $\Delta = 16 - 4k^2 > 0 \Rightarrow k^2 < 4$ M1
2. **$-2 < k < 2$, and $k \neq 0$ (otherwise it is not a quadratic)** A1 R1

EXAM TIP

"Line meets curve" questions: set them equal, rearrange to = 0, then use Δ. Tangent $\Leftrightarrow \Delta = 0$; meets twice $\Leftrightarrow \Delta > 0$; misses $\Leftrightarrow \Delta < 0$.

4 Quadratic inequalities

WORKED EXAMPLE

Solve $x^2 - x - 6 < 0$.

1. $(x - 3)(x + 2) < 0$ M1
2. the graph is below the x-axis between its roots R1
3. **$-2 < x < 3$** A1

COMMON MISTAKE

Writing "x < 3 and x < −2". Sketch the parabola: for "< 0" the answer is between the roots; for "> 0" it is outside them (x < −2 or x > 3).

5 Equations that reduce to quadratics

WORKED EXAMPLE

Solve $x^4 - 5x^2 + 4 = 0$.

1. let $u = x^2$: $u^2 - 5u + 4 = 0 \Rightarrow u = 1$ or 4 M1 A1
2. **$x = \pm 1, \pm 2$** A1

WORKED EXAMPLE

Solve $2^{2x} - 6(2^x) + 8 = 0$.

1. let $u = 2^x$: $u^2 - 6u + 8 = 0 \Rightarrow u = 2$ or 4 M1
2. **$2^x = 2$ or 4, so $x = 1$ or 2** A1

6 Quadratic models

WORKED EXAMPLE

A ball's height is $h = 20t - 5t^2$ (metres, t in seconds). Find the maximum height and when it lands.

1. vertex at $t = -20/(2 \times (-5)) = 2$ s M1
2. $h = 40 - 20 =$ **20 m** A1
3. **lands when $20t - 5t^2 = 0 \Rightarrow t = 4$ s ($t = 0$ is the launch)** A1

GDC (CALCULATOR) METHOD

Paper 2: graph the quadratic and use zero and maximum/minimum, or the polynomial solver. Exact answers (surds) still need the formula.

LINKING QUESTION

The discriminant reappears whenever a line meets a curve (tangents), and in complex numbers (HL), where $\Delta < 0$ gives a conjugate pair of complex roots.

IDEA FOR YOUR IA / EXPLORATION

Model a real parabola: a fountain or a basketball shot from a video (with free tracking software), fit $y = ax^2 + bx + c$ and compare the fitted vertex with the measurement.

Test yourself — cover the answers first

QUICK CHECK

1. State the axis of symmetry of $y = 2x^2 - 8x + 1$.
2. Write $x^2 + 10x$ in completed-square form.
3. What does $\Delta < 0$ mean for the graph?
4. Solve $x^2 = 5x$.
5. Find the roots of $(x - 4)(2x + 1) = 0$.

EXAM-STYLE

6. The quadratic $f(x) = x^2 - 2kx + (k + 6)$ has no real roots. Find the range of k.
7. A rectangle has perimeter 40 cm. Find the dimensions that give the maximum area and that area.

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Show answers

1. $x = 2$
2. $(x + 5)^2 - 25$
3. it does not cross or touch the x-axis
4. $x = 0$ or $x = 5$
5. $x = 4$, $x = -½$
6. $\Delta = 4k^2 - 4(k + 6) < 0$
   $k^2 - k - 6 < 0 \Rightarrow (k - 3)(k + 2) < 0$
   $-2 < k < 3$
7. sides x and $20 - x$: $A = x(20 - x)$
   vertex at $x = 10$
   10 cm by 10 cm, area 100 cm^2

Stuck on a question?

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Want regular help? Send this to your parent →

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