IB DP MATHS AA · TOPIC 2 FUNCTIONS · SL 2.11 · AHL 2.16 · ~6 MIN READ

Transformations of graphs

Translations, reflections and stretches of y = f(x), combining transformations in the right order, mapping points, and (HL) the graphs of |f(x)|, f(|x|), 1/f(x), f(ax + b) and [f(x)]².

Translations | Reflections | Stretches | Combined transformations | Mapping points | |f(x)| & f(|x|) (HL) | 1/f(x) (HL) | [f(x)]² (HL)

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

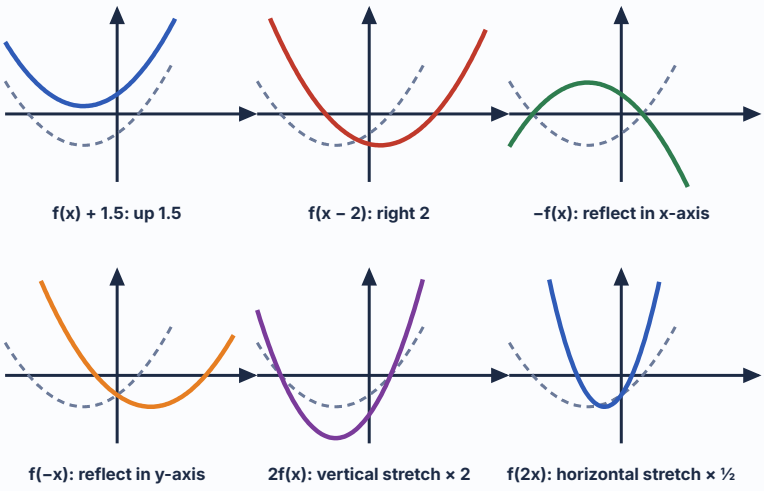
IN 30 SECONDS

- **Outside the bracket changes y** (as you would expect): f(x) + b moves up b, pf(x) stretches vertically by p, −f(x) reflects in the x-axis.
- **Inside the bracket changes x** (the opposite way): f(x − a) moves RIGHT a, f(qx) stretches horizontally by **1/q**, f(−x) reflects in the y-axis.
- **Combined:** do the inside changes in the order a point's x-value would be undone, and the outside changes in the order you would calculate them. Tracking one point is the safest check.
- **HL:** |f(x)| flips negative parts up; f(|x|) mirrors the right half; 1/f(x) turns roots into asymptotes; $[f(x)]^2$ is never negative.

BY THE END YOU CAN

- Describe and sketch translations, reflections and stretches of y = f(x).
- Apply combined transformations and find the image of a point.
- Write down the equation of a transformed graph from a description.
- Sketch y = |f(x)|, y = f(|x|), y = 1/f(x), y = f(ax + b) and y = $[f(x)]^2$. HL

1 The six basic transformations



Grey dashed: the original y = f(x).

Each panel changes the grey curve in one way.

New graph	Transformation	Point (x, y) goes to
$y = f(x) + b$	translation by (0, b)	$(x, y + b)$
$y = f(x - a)$	translation by (a, 0)	$(x + a, y)$
$y = pf(x)$	vertical stretch, factor p	(x, py)
$y = f(qx)$	horizontal stretch, factor 1/q	$(x/q, y)$
$y = -f(x)$	reflection in the x-axis	$(x, -y)$
$y = f(-x)$	reflection in the y-axis	$(-x, y)$

COMMON MISTAKE

Thinking $f(x + 3)$ moves the graph right. $f(x + 3)$ moves it **left** 3: the new graph reaches each y-value when x is 3 smaller.

2 Combining transformations

WORKED EXAMPLE

Describe the transformations that map $y = f(x)$ onto $y = 2f(x - 3) + 1$.

1. horizontal translation 3 right (inside) A1
2. **vertical stretch factor 2, then vertical translation 1 up (outside: stretch before shifting)** A1 A1

WORKED EXAMPLE

The point (4, 6) lies on $y = f(x)$. Find its image on $y = 3f(2x) - 1$.

1. x: $2x = 4 \Rightarrow x = 2$ M1
2. y: $3 \times 6 - 1 = 17$ M1
3. **(2, 17)** A1

WORKED EXAMPLE

$y = x^2$ is translated by (2, −3) and then reflected in the x-axis. Find the equation.

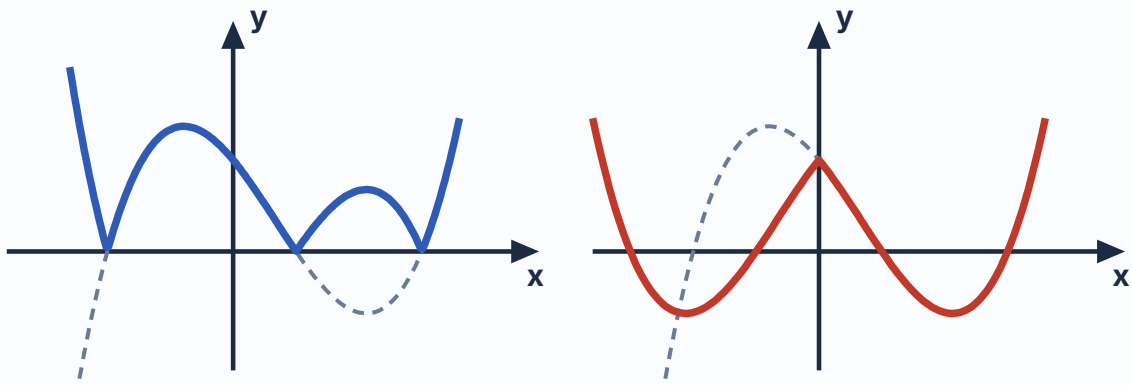
1. translation: $y = (x - 2)^2 - 3$ A1
2. **reflection: $y = -(x - 2)^2 + 3$** A1

EXAM TIP

The order matters for outside changes: $2f(x) + 1$ (stretch then add) is not $2(f(x) + 1) = 2f(x) + 2$. Track one point through each step to check.

- **Asymptotes move too:** if $y = f(x)$ has asymptote $x = 1$, then $y = f(x - 2)$ has $x = 3$; $y = f(x) + 4$ moves a horizontal asymptote up 4.

3 Modulus graphs HL

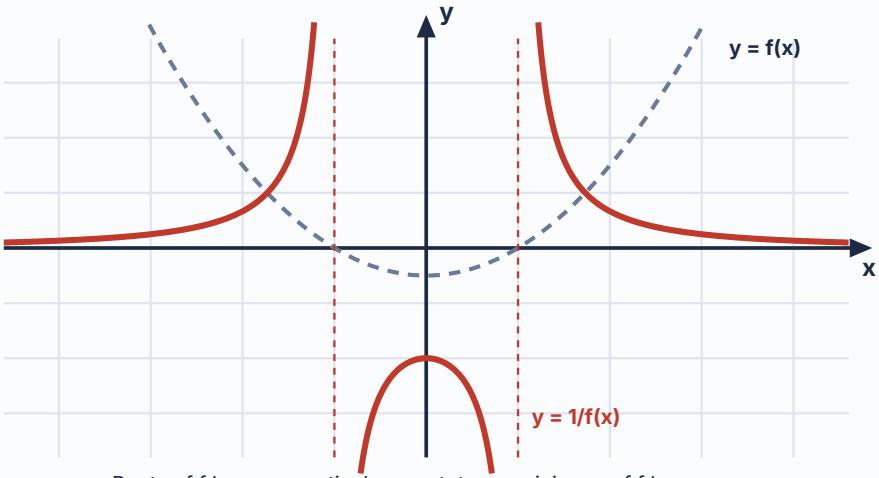


|f(x)|: flip the parts below the x-axis **f(|x|): keep x ≥ 0, mirror it**

Left: reflect below-axis parts upwards. Right: throw away x < 0 and copy x ≥ 0 across the y-axis.

- **y = |f(x)|:** keep parts with y ≥ 0; reflect parts with y < 0 in the x-axis. Points on the x-axis stay put and often become sharp corners.
- **y = f(|x|):** for x ≥ 0 it is the same as f(x); for x < 0 it mirrors the x ≥ 0 part. The graph is always even.
- **y = f(ax + b):** write it as f(a(x + b/a)): a horizontal stretch by 1/a, then a translation of −b/a.

4 Reciprocal and squared graphs HL



Roots of f become vertical asymptotes; a minimum of f becomes a maximum of 1/f; and 1/f → 0 where f → ±∞.

Every root of f gives a vertical asymptote of 1/f, and f and 1/f cross where f = ±1.

- **$y = 1/f(x)$:** roots of f → vertical asymptotes; as f → ±∞, 1/f → 0; a maximum of f becomes a minimum of 1/f (and vice versa) at the same x; 1/f has the same sign as f.
- **$y = [f(x)]^2$:** always ≥ 0; roots of f become touching points (minima); where $|f| < 1$ the graph lies closer to the axis, where $|f| > 1$ it rises above $|f|$.

WORKED EXAMPLE

$f(x) = x^2 - 4$. Sketch $y = 1/f(x)$: give asymptotes and the turning point.

1. vertical asymptotes at the roots: **$x = \pm 2$** A1
2. horizontal asymptote **$y = 0$** A1
3. **f has a minimum at (0, −4), so 1/f has a maximum at (0, −¼)** A1

GDC (CALCULATOR) METHOD

Check every sketch on the GDC with abs( ) or 1/( ), but in Paper 1 you must be able to draw them from the rules above.

LINKING QUESTION

Trig graphs are transformations too: $y = 3\sin(2(x - \pi/4)) + 1$ is a stretch, a stretch, a shift and a shift of sin x (Trig identities, equations and graphs).

IDEA FOR YOUR IA / EXPLORATION

Explore how families of transformed graphs model real data, for example fitting $y = a\sin(b(x - c)) + d$ to daylight hours or tide heights.

Test yourself — cover the answers first

QUICK CHECK

1. Describe $y = f(x) + 5$.
2. Describe $y = f(x + 2)$.
3. Describe $y = f(3x)$.
4. (2, 5) is on $y = f(x)$. Where is it on $y = -f(x)$?
5. HL What is the image of the root $x = 3$ of f on $y = 1/f(x)$?

EXAM-STYLE

6. The graph of $y = f(x)$ has a maximum at (1, 4). Find the maximum of $y = 2f(x + 3) - 5$.
7. HL $f(x) = x(x - 3)$. Find the coordinates of the turning point of $y = |f(x)|$ between $x = 0$ and $x = 3$.

Show answers

1. translation up 5
2. translation 2 left
3. horizontal stretch, factor ⅓
4. (2, −5)
5. a vertical asymptote $x = 3$
6. x: 1 − 3 = −2
   y: 2(4) − 5 = 3
   (−2, 3)
7. f has a minimum at (1.5, −2.25)
   reflected upwards
   (1.5, 2.25), a maximum of $|f|$

Stuck on a question?

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