IB DP MATHS AA · TOPIC 3 GEOMETRY AND TRIGONOMETRY · SL 3.1–3.4 · ~7 MIN READ

Trigonometry: triangles, radians, arcs and sectors

Distance and midpoint in 3D, volume and surface area of solids, angles in 3D, right-angled trig, the sine rule (including the ambiguous case), the cosine rule, area ½ab sin C, bearings, radians, arc length and sector area.

3D distance & midpoint | Volume & surface area | Angles in 3D | Sine rule | Ambiguous case | Cosine rule | Area ½ab sin C | Bearings

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Right-angled triangles:** SOH CAH TOA. **Any triangle:** sine rule for "pairs" (a side and its opposite angle known), cosine rule for SAS or SSS.
- **Area of any triangle = ½ab sin C** (two sides and the angle between them).
- **The ambiguous case:** given two sides and a non-included angle, sin B = k can give **two** angles, B and 180° − B. Check both.
- **Radians:** π rad = 180°. With θ in radians, **arc length = rθ** and **sector area = ½r^2θ**.
- This note is the same at SL and HL, and unchanged for 2029.

BY THE END YOU CAN

- Find distances, midpoints, volumes and surface areas of 3D solids.
- Find angles between lines and between a line and a plane in 3D.
- Use the sine rule, including the ambiguous case, and the cosine rule.
- Find the area of a triangle with ½ab sin C, and solve bearings problems.
- Convert between degrees and radians and find arc lengths, sector areas and segment areas.

Riyaz Sir

1 3D geometry

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$ FORMULA BOOKLET

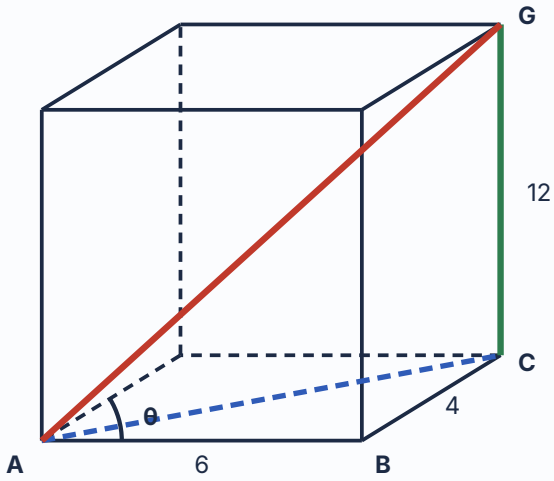
distance between two points in 3D

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$ FORMULA BOOKLET

average each coordinate

Solid	Volume	Surface area
Sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
Cone	$\frac{1}{3}\pi r^2 h$	$\pi r l$ (curved), l = slant height
Pyramid	$\frac{1}{3}$ × base area × h	add the faces
Prism / cylinder	base area × h	$2\pi r h + 2\pi r^2$ (cylinder)

- All of these are in the formula booklet. Hemispheres and composite solids come up often: add or subtract the pieces, and remember the flat face of a hemisphere.



angle between AG
and the base ABCD:
tan θ = CG/AC
AC: base diagonal (blue)
CG: height (green)

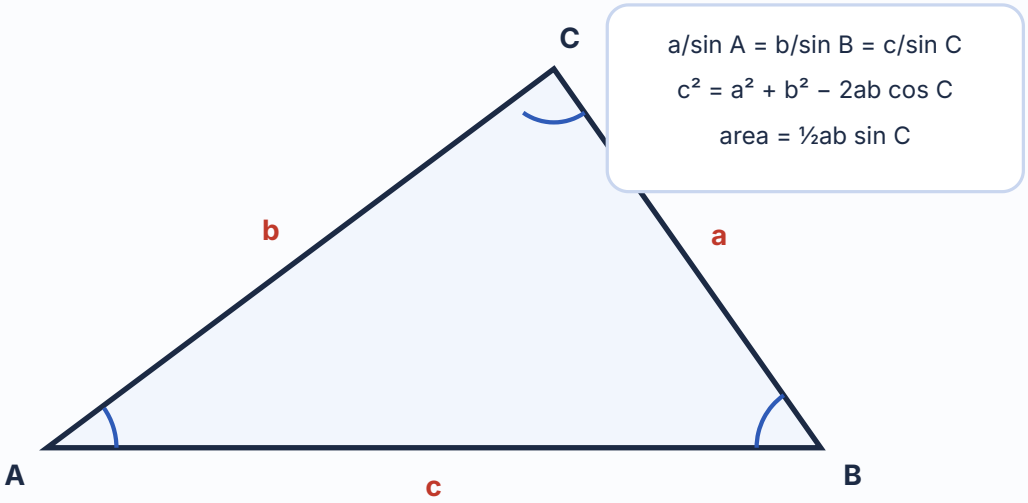
To find the angle between a line and a plane, drop a perpendicular to the plane and use the right-angled triangle that forms.

WORKED EXAMPLE

A cuboid has base 6 cm by 4 cm and height 12 cm. Find the angle between the diagonal AG and the base.

1. base diagonal AC = $\sqrt{(6^2 + 4^2)} = \sqrt{52}$ M1
2. $\tan\theta = 12/\sqrt{52}$ M1
3. **$\theta = 59.0°$** A1

2 Sine rule, cosine rule and area



$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

$c^2 = a^2 + b^2 - 2ab\cos C$

area $= \frac{1}{2}ab\sin C$

Side a is opposite angle A, and so on. All three results are in the formula booklet.

Label sides with lower-case letters opposite the matching capital-letter angles.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$ FORMULA BOOKLET

sine rule: use with a side–angle pair

$$c^2 = a^2 + b^2 - 2ab\cos C,\ \cos C = \frac{a^2 + b^2 - c^2}{2ab}$$ FORMULA BOOKLET

cosine rule: SAS or SSS

$$A = \tfrac{1}{2}ab\sin C$$ FORMULA BOOKLET

area of a triangle

WORKED EXAMPLE

In triangle ABC, A = 40°, B = 65° and a = 8 cm. Find b.

1. $\frac{b}{\sin 65°} = \frac{8}{\sin 40°}$ M1
2. **b = 11.3 cm** A1

WORKED EXAMPLE

In triangle ABC, a = 7 cm, b = 9 cm and C = 50°. Find c and the area.

1. $c^2 = 49 + 81 - 126 \cos 50°$ **M1**
2. **c = 7.00 cm** **A1**
3. **area = ½(7)(9) sin 50° = 24.1 cm^2** **A1**

WORKED EXAMPLE

A triangle has sides 5, 7 and 9 cm. Find its largest angle.

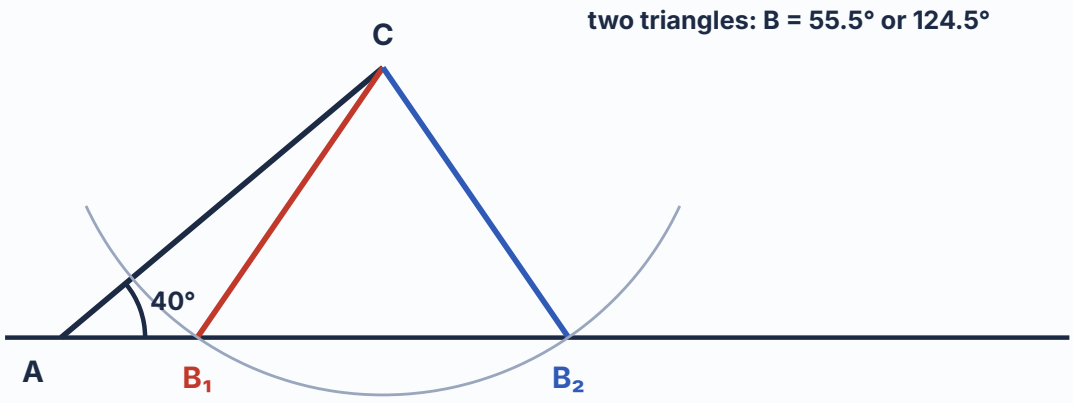
1. the largest angle is opposite the longest side **R1**
2. $\cos\theta = \frac{25 + 49 - 81}{2 \times 5 \times 7} = -0.1$ **M1**
3. **θ = 95.7°** **A1**

COMMON MISTAKE

Using the sine rule to find the largest angle of a triangle. The sine rule cannot tell 84° from 96°; the cosine rule can, because cos is negative for obtuse angles.

3 The ambiguous case

Given A, b and a (SSA): side a can swing to two positions



With angle A and sides b and a fixed, side a can meet the base in two places.

WORKED EXAMPLE

In triangle ABC, A = 40°, a = 11.7 cm and b = 15 cm. Find the possible values of B.

1. $\sin B = \frac{15 \sin 40°}{11.7} = 0.824$ M1
2. $B = 55.5°$ A1
3. **or $B = 180° - 55.5° = 124.5°$; both work because $124.5° + 40° < 180°$** A1 R1

EXAM TIP

Two triangles are possible only when the given angle is opposite the **shorter** of the two given sides. If it is opposite the longer side, there is just one triangle.

4 Bearings

- **Bearings** are measured clockwise from North and written with three figures: 060°, 150°, 305°.

WORKED EXAMPLE

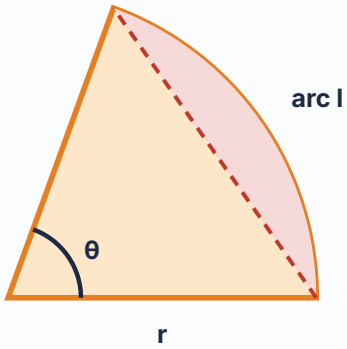
A ship sails 12 km on a bearing of 060°, then 8 km on a bearing of 150°. Find its distance and bearing from the start.

1. the turn is $150° - 60° = 90°$, so the triangle is right-angled R1
2. distance = $\sqrt{(144 + 64)}$ = **14.4 km** A1
3. **angle at the start = $\tan^{-1}(8/12) = 33.7°$, so bearing = $60° + 33.7° = 094°$** M1 A1

EXAM TIP

Always draw a North line at every point. Co-interior angles between parallel North lines add to 180°, which gives the angle inside the triangle.

5 Radians, arcs and sectors



θ in radians

arc length $l = r\theta$

sector area = $\frac{1}{2}r^2\theta$

segment (red) = $\frac{1}{2}r^2(\theta - \sin\theta)$

π rad = 180°

In radians the formulas lose their "÷ 360" and become short.

$l = r\theta$, $A = \frac{1}{2}r^2\theta$ **FORMULA BOOKLET**

θ in radians

- **Converting:** multiply degrees by π/180; multiply radians by 180/π. Learn 30° = π/6, 45° = π/4, 60° = π/3, 90° = π/2.

WORKED EXAMPLE

A sector has radius 5 cm and angle 0.8 rad. Find its perimeter and area.

1. arc = 5 × 0.8 = 4 cm **A1**
2. perimeter = 4 + 5 + 5 = **14 cm** **A1**
3. **area = ½(25)(0.8) = 10 cm^2** **A1**

WORKED EXAMPLE

Find the area of the segment cut off by a chord that subtends π/3 at the centre of a circle of radius 6 cm.

1. sector − triangle = ½(36)(π/3) − ½(36) sin(π/3) **M1**
2. = 6π − 9√3 **A1**
3. **= 3.26 cm^2** **A1**

COMMON MISTAKE

Leaving the calculator in degree mode when a question is in radians (or the reverse). In Paper 1 and 2, check the mode at the start of every trig question.

GDC (CALCULATOR) METHOD

Paper 2: the GDC solves triangles fast, but write the rule you used and the substituted values first, or you lose the method marks.

LINKING QUESTION

The unit circle extends sin, cos and tan to all angles (Trig identities, equations and graphs). Radians are needed for calculus, because d/dx (sin x) = cos x only in radians (Differentiation).

IDEA FOR YOUR IA / EXPLORATION

Use trigonometry to measure something you can't reach (the height of a building from two angles of elevation) and compare with the real value, or model the geometry of a sports pitch.

Test yourself — cover the answers first

QUICK CHECK

1. Convert 150° to radians.
2. Find the arc length for r = 10 cm, θ = 0.3 rad.
3. Which rule finds an angle from three sides?
4. Find the area of a triangle with sides 4 and 6 cm and an included angle of 30°.
5. When can the ambiguous case happen?

EXAM-STYLE

6. A sector of radius r and angle θ has perimeter 20 cm and area 16 cm². Find r and θ.
7. Points A(1, 2, −1) and B(5, −2, 1). Find AB and the midpoint of AB.

Show answers

1. 5π/6
2. 3 cm
3. the cosine rule
4. 6 cm²
5. two sides and a non-included angle are given
6. $2r + r\theta = 20$ and $\frac{1}{2}r^2\theta = 16$
   $r\theta = 20 - 2r$, so $\frac{1}{2}r(20 - 2r) = 16 \Rightarrow r^2 - 10r + 16 = 0$
   r = 2, θ = 8 or **r = 8, θ = 0.5** (θ = 8 rad is more than 2π, so reject it if θ must be less than 2π)
7. AB = √(16 + 16 + 4) = **6**
   midpoint **(3, 0, 0)**

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

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