IB DP MATHS AI · TOPIC 1 NUMBER AND ALGEBRA · SL 1.1, 1.6 · ~6 MIN READ

Approximation, standard form and errors

Standard form and calculator E-notation, rounding to decimal places and significant figures, upper and lower bounds of measurements and calculations, estimation, and percentage error, all set in the real contexts IB Maths AI uses.

Standard form $a \times 10^k$ | GDC E-notation | Decimal places & significant figures | Upper and lower bounds | Bounds of calculations | Estimation | Percentage error

RS by Riyaz Sir
mentorriyaz.in

No tag = **SL and HL** | HL ONLY HL students only | FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded | highlight = comes up again and again | Test yourself at the end

IN 30 SECONDS

- **Standard form:** $a \times 10^k$ with $1 \le a < 10$ and k an integer. Your GDC writes 1.28E4 for 1.28×10^4, but you must write 1.28×10^4 in your answer.
- **Rounding:** decimal places count after the point; significant figures count from the first non-zero digit. IB's default is **3 s.f.** unless the question says otherwise.
- **Bounds:** a measurement rounded to the nearest unit lies within half a unit either way. 12.4 cm (1 d.p.) means $12.35 \le L < 12.45$.
- **Percentage error** $\varepsilon = \frac{|v_A - v_E|}{|v_E|} \times 100\%$, where v_A is the approximate value and v_E the exact value.
- This note is the same at SL and HL, and unchanged for 2029.

BY THE END YOU CAN

- Write and calculate with numbers in standard form, and read GDC E-notation.
- Round to a given number of decimal places or significant figures.
- Find upper and lower bounds of rounded measurements and of calculations that use them.
- Estimate answers and find percentage errors.

1 Standard form

- **Big numbers:** 384 000 km = 3.84×10^5 km. **Small numbers:** 0.000 052 m = 5.2×10^{-5} m.

WORKED EXAMPLE

Calculate $(3.2 \times 10^5) \times (4 \times 10^{-2})$ and give the answer in standard form.

1. $3.2 \times 4 = 12.8$ and $10^5 \times 10^{-2} = 10^3$ M1
2. **$12.8 \times 10^3 = 1.28 \times 10^4$** A1

WORKED EXAMPLE

Light travels at 3×10^5 km s^{-1}. The Sun is 1.496×10^8 km from Earth. How many minutes does sunlight take to reach Earth?

1. time $= \dfrac{1.496 \times 10^8}{3 \times 10^5} = 498.67$ s M1 A1
2. **≈ 8.31 minutes** A1

COMMON MISTAKE

Writing a GDC answer such as 1.28E4 as your final answer. Examiners don't accept calculator notation: write 1.28×10^4.

GDC (CALCULATOR) METHOD

Enter powers of ten with the EE (or ×10ⁿ) key, not "× 10 ^". Using brackets avoids order-of-operations slips with negative powers.

2 Rounding and significant figures

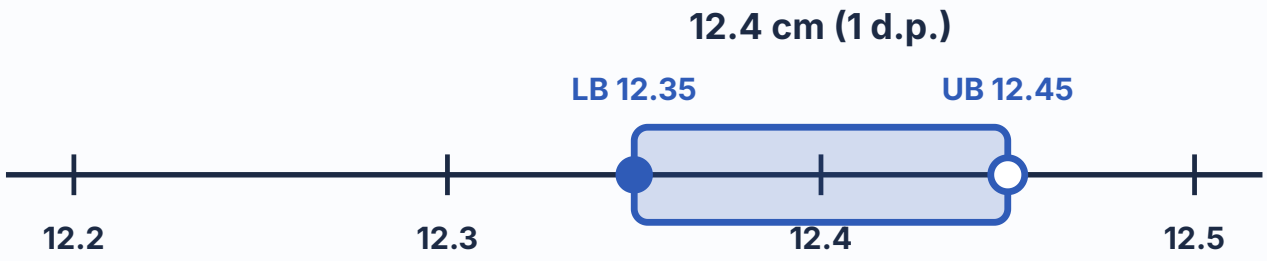
Number	3 s.f.	2 d.p.
4567.891	4570	4567.89
0.003 0456	0.003 05	0.00
12.0049	12.0	12.00

- **Zeros:** zeros between non-zero digits count (2005 has 4 s.f.); leading zeros never count (0.004 has 1 s.f.).
- **Exam rule:** give exact answers or answers correct to 3 s.f. unless told otherwise. A wrong level of accuracy can lose a mark, so **keep full values on the GDC** and round only at the end.

COMMON MISTAKE

Rounding in the middle of a calculation. Rounding 498.67 s to 499 s and then dividing by 60 still gives 8.32, which is wrong at 3 s.f. Store values in memory instead.

3 Upper and lower bounds



Every length from 12.35 up to (but not including) 12.45 rounds to 12.4: $12.35 \le L < 12.45$

The lower bound is included; the upper bound is the edge of the interval and is not.

- **Rule:** if a value is rounded to the nearest u, the bounds are value ± ½u.

WORKED EXAMPLE

A rectangle is 12.4 cm by 5.0 cm, both correct to 1 d.p. Find the upper and lower bounds of its area.

1. length 12.35 to 12.45, width 4.95 to 5.05 **A1**
2. upper bound = 12.45 × 5.05 = **62.9 cm^2** (62.8725) **M1** **A1**
3. **lower bound = 12.35 × 4.95 = 61.1 cm^2 (61.1325)** **A1**

- **Dividing:** the upper bound of a ÷ b is (upper a) ÷ (lower b); the lower bound is (lower a) ÷ (upper b).

WORKED EXAMPLE

A runner covers 100 m (to the nearest metre) in 12.3 s (to the nearest 0.1 s). Find the bounds of her average speed.

1. upper = $\frac{100.5}{12.25}$ = **8.20 m s^{-1}** **M1** **A1**
2. **lower = $\frac{99.5}{12.35}$ = 8.06 m s^{-1}** **A1**

EXAM TIP

Subtraction works the same way: the largest a − b is (upper a) − (lower b). Write out which bound you use for each value before you calculate.

4 Estimation and percentage error

$$\varepsilon = \left|\frac{v_A - v_E}{v_E}\right| \times 100\%$$ FORMULA BOOKLET

percentage error; divide by the exact value

WORKED EXAMPLE

Estimate $\frac{\sqrt{48.7} \times 19.6}{0.52}$ by rounding each number, then find the percentage error of your estimate.

1. estimate $\approx \frac{7 \times 20}{0.5} = 280$ M1 A1
2. exact value = 263.04 (GDC) A1
3. $\varepsilon = \frac{|280 - 263.04|}{263.04} \times 100\% = 6.45\%$ M1 A1

WORKED EXAMPLE

A student uses $\frac{22}{7}$ for π. Find the percentage error.

1. $\varepsilon = \frac{|22/7 - \pi|}{\pi} \times 100\%$ M1
2. **= 0.0402%** A1

WORKED EXAMPLE

A crowd is estimated as 50 000. The true number is 47 300. Find the percentage error.

1. $\varepsilon = \frac{|50\,000 - 47\,300|}{47\,300} \times 100\%$ M1
2. **= 5.71%** A1

COMMON MISTAKE

Dividing by the approximate value. The formula always divides by the **exact** value v_E.

- **Why estimate?** Estimating first catches GDC slips: if your estimate is 280 and the GDC says 2630, you have mistyped something.

LINKING QUESTION

Bounds and errors return in Statistics, correlation and regression (how accurate is a prediction?) and in Integration, areas and kinematics (how good is the trapezoidal rule?). Standard form appears in Exponents, logarithms and equations, for example in pH and decibel scales.

IDEA FOR YOUR IA / EXPLORATION

Measure something many times (the period of a pendulum, the height of classmates) and analyse how rounding and measurement error affect a calculated result, such as g from a pendulum.

Test yourself — cover the answers first

QUICK CHECK

1. Write 0.000 708 in standard form.
2. Round 0.049 96 to 3 s.f.
3. A mass is 64 kg to the nearest kg. Give its bounds.
4. What does the GDC display 3.5E−3 mean?
5. Find the percentage error when 9.8 is used for 9.81.

EXAM-STYLE

6. A square has side 8 cm, correct to the nearest cm. Find the upper and lower bounds of its perimeter and of its area.
7. The population of a city is 1.84×10^7 and its area is 4.35×10^3 km^2. Find the population density, and the percentage error if the density is estimated as 4000 people per km^2.

Show answers

1. 7.08×10^{-4}
2. 0.0500
3. $63.5 \leq m < 64.5$ kg
4. 3.5×10^{-3}
5. 0.102%
6. side 7.5 to 8.5 cm
   perimeter **$30 \leq P < 34$ cm**
   area **$56.25 \leq A < 72.25$ cm²**
7. density $= \dfrac{1.84 \times 10^{7}}{4.35 \times 10^{3}} = 4229.9 \approx$ **4230 people per km²**
   $\varepsilon = \dfrac{|4000 - 4229.9|}{4229.9} \times 100\% =$ **5.43%**

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

Share with a friend

Ask Riyaz Sir