IB DP MATHS AI · TOPIC 1 NUMBER AND ALGEBRA · AHL 1.12, 1.13 · ~6 MIN READ

Complex numbers

HL complex numbers for Maths AI: Cartesian form and the Argand diagram, modulus and argument, polar and Euler forms, multiplying and dividing, solving quadratics with complex roots, and adding sinusoidal waves (phasors) with complex numbers.

$i^2 = -1$ · Cartesian form a + bi · Argand diagram · Modulus & argument · Polar & Euler form · Multiplying & dividing · Powers · Complex roots of quadratics

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No tag = **SL and HL** · HL ONLY HL students only · FORMULA BOOKLET given in the exam
M1 A1 how marks are awarded · highlight = comes up again and again · Test yourself at the end

IN 30 SECONDS

- A **complex number** z = a + bi has real part a and imaginary part b, where $i^2 = -1$. Plot it as the point (a, b) on an **Argand diagram**.
- **Modulus** $r = |z| = \sqrt{(a^2 + b^2)}$; **argument** θ = arg z, the angle from the positive real axis (in radians, usually $-\pi < \theta \leq \pi$).
- **Three forms:** $z = a + bi = r(\cos\theta + i\sin\theta) = re^{i\theta}$. Multiply by multiplying moduli and **adding** arguments; divide by dividing moduli and subtracting arguments.
- **Why AI cares:** waves with the same frequency (AC voltages, sound) add like complex numbers. This turns a messy trig sum into one arrow addition.

BY THE END YOU CAN

- Calculate with complex numbers in Cartesian form, including conjugates and division. HL
- Represent complex numbers on an Argand diagram and find modulus and argument. HL
- Convert between Cartesian, polar and Euler forms. HL
- Multiply, divide and find powers in polar and Euler form, and interpret them geometrically. HL
- Solve quadratic equations with complex roots. HL
- Use complex numbers to add sinusoidal functions with the same frequency. HL

SYLLABUS NOTE

From first exams in 2029, complex numbers leave the AI HL syllabus. If you sit exams up to November 2028, you still need everything in this note.

1 Cartesian form HL

- **Adding:** add real parts and imaginary parts. **Multiplying:** expand the brackets and replace i^2 with −1.
- **Conjugate:** $z^* = a - bi$. $z z^* = a^2 + b^2$ is always real, which is how you divide.

WORKED EXAMPLE

$z = 3 + 4i$ and $w = 1 - 2i$. Find zw and $\frac{z}{w}$.

1. $zw = 3 - 6i + 4i - 8i^2 = \mathbf{11 - 2i}$ M1 A1
2. $\frac{z}{w} = \frac{(3 + 4i)(1 + 2i)}{(1 - 2i)(1 + 2i)} = \frac{-5 + 10i}{5}$ M1
3. $\mathbf{= -1 + 2i}$ A1

WORKED EXAMPLE

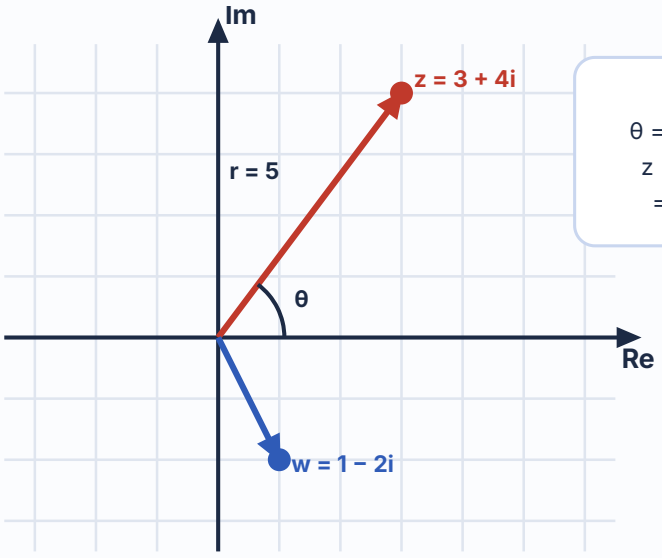
Solve $z^2 + 4z + 13 = 0$.

1. $z = \frac{-4 \pm \sqrt{(16 - 52)}}{2} = \frac{-4 \pm \sqrt{(-36)}}{2}$ M1
2. $\mathbf{z = -2 \pm 3i}$ A1

EXAM TIP

Quadratics with real coefficients have complex roots in **conjugate pairs**: if $2 + 5i$ is a root, so is $2 - 5i$.

2 Modulus, argument and the Argand diagram HL



r = |z| = 5
θ = arg z = 0.927
z = 5e^(0.927i)
= 5 cis 0.927

The modulus is the length of the arrow; the argument is its angle from the positive real axis.

$|z| = \sqrt{(a^2 + b^2)}$, $\arg z = \theta$ with $\tan\theta = \frac{b}{a}$ FORMULA BOOKLET

check the quadrant

$$z = r(\cos\theta + i\sin\theta) = re^{i\theta} = r\,\text{cis}\,\theta$$ FORMULA BOOKLET

polar and Euler forms

WORKED EXAMPLE

Write z = 3 + 4i in polar and Euler form.

1. $r = \sqrt{(9 + 16)} = 5$ A1
2. $\theta = \arctan(\frac{4}{3}) = 0.927$ (first quadrant) A1
3. **$z = 5(\cos 0.927 + i \sin 0.927) = 5e^{0.927i}$** A1

COMMON MISTAKE

Trusting arctan for every quadrant. For $z = -1 - i$, $\arctan(1) = \pi/4$, but z is in the third quadrant, so $\arg z = -\frac{3\pi}{4}$. Sketch the point first.

WORKED EXAMPLE

Write $2e^{i\pi/3}$ in Cartesian form.

1. $2(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$ M1
2. **$= 1 + \sqrt{3}\,i$** A1

3 Multiplying, dividing and powers HL

$$r_1e^{i\theta_1} \times r_2e^{i\theta_2} = r_1r_2e^{i(\theta_1 + \theta_\{2\})}$$

multiply moduli, add arguments

- **Geometry:** multiplying by i rotates a point 90° anticlockwise about the origin. Multiplying by $re^{i\theta}$ stretches by r and rotates by θ.
- **Powers:** $(re^{i\theta})^n = r^ne^{in\theta}$ (De Moivre's theorem).

WORKED EXAMPLE

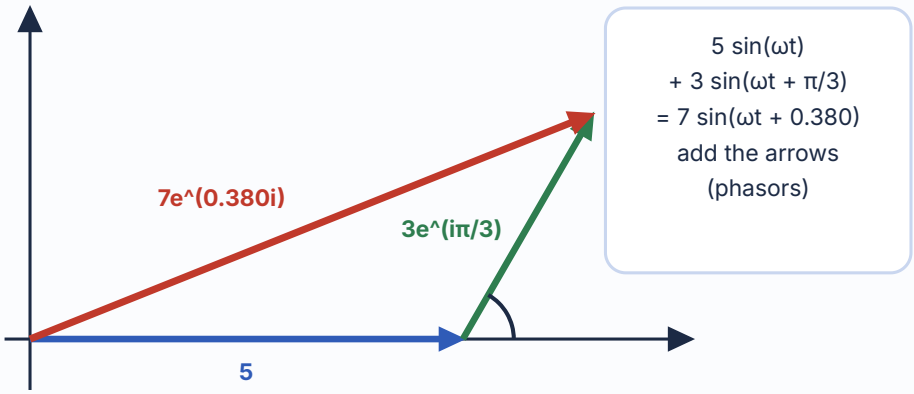
Find $(1 + i)^4$.

1. $1 + i = \sqrt{2}\,e^{i\pi/4}$ M1
2. $(\sqrt{2})^4\,e^{i\pi} = 4e^{i\pi}$ M1
3. **$= -4$** A1

GDC (CALCULATOR) METHOD

Most GDCs work in complex mode: set it to a + bi or $re^{\theta i}$, and use the abs( ) and angle( ) functions. Use radians.

4 Adding sinusoidal waves HL



Waves of the same frequency add like complex numbers: amplitudes and phases come out together.

Represent each wave by an arrow (a phasor), add the arrows, then read off the length and angle.

- **Method:** write a $\sin(\omega t + \alpha)$ as the complex number $ae^{i\alpha}$. Add the complex numbers. The modulus is the new amplitude and the argument is the new phase shift.

WORKED EXAMPLE

Two AC voltages are $V_1 = 5\sin(\omega t)$ and $V_2 = 3\sin(\omega t + \frac{\pi}{3})$. Write $V_1 + V_2$ in the form $R\sin(\omega t + \varphi)$.

1. $5 + 3e^{i\pi/3} = 5 + 1.5 + 2.598i = 6.5 + 2.598i$ M1 A1
2. $R = |6.5 + 2.598i| = 7$ A1
3. **$\varphi = \arg = 0.380$, so $V = 7\sin(\omega t + 0.380)$** A1

COMMON MISTAKE

Adding amplitudes directly: 5 + 3 = 8. The waves are out of phase, so their peaks don't line up and the combined amplitude is smaller (here 7).

LINKING QUESTION

The argument uses the radian measure and trig values from Radians, vectors and matrix transformations, and rotation by multiplying complex numbers matches rotation matrices. Maths AA covers complex numbers with more algebra and proof: see Complex numbers.

IDEA FOR YOUR IA / EXPLORATION

Model the sound of two tuning forks or an AC circuit by adding sinusoids with complex numbers, and compare the predicted amplitude with a measurement from a phone sound app or an oscilloscope simulation.

Test yourself — cover the answers first

QUICK CHECK

1. HL Simplify i^3.
2. HL Find $(2 + i)(2 - i)$.
3. HL Find $|-5 + 12i|$.
4. HL What is the argument of $z = 4i$?

5. HL Write $3e^{i\pi}$ in Cartesian form.

EXAM-STYLE

6. HL $z = 2e^{i\pi/6}$ and $w = 3e^{i\pi/3}$. Find zw in Euler and Cartesian form, and describe the effect of multiplying by w.
7. HL Write $4\cos(t) + 4\sin(t)$ in the form $R\sin(t + \varphi)$.

Show answers

1. $-i$
2. 5
3. 13
4. $\pi/2$
5. -3
6. $zw = 6e^{i\pi/2} = \mathbf{6i}$

   multiplying by w stretches by a factor of 3 and rotates by $\frac{\pi}{3}$ anticlockwise
7. $4\sin t \leftrightarrow 4$, and $4\cos t = 4\sin(t + \pi/2) \leftrightarrow 4i$

   $4 + 4i = 4\sqrt{2}\, e^{i\pi/4}$

   $\mathbf{= 4\sqrt{2}\sin(t + \frac{\pi}{4})}$

Stuck on a question?

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