IB DP MATHS AI · TOPIC 1 NUMBER AND ALGEBRA · SL 1.5, 1.8 · AHL 1.9, 1.10 · ~6 MIN READ

Exponents, logarithms and equations

Laws of exponents, an introduction to logarithms with base 10 and e, log scales such as pH and decibels, solving systems of linear equations and polynomial equations with the GDC, and for HL: the laws of logarithms and rational exponents.

Laws of exponents | Logarithms base 10 and e | Solving $a^x = b$ | pH and decibel scales | Systems of linear equations | Polynomial equations | Laws of logarithms (HL) | Rational exponents (HL)

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam
M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Exponent laws:** $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$.
- **A logarithm is an exponent:** $a^x = b \Leftrightarrow x = \log_a b$. Your GDC has **log** (base 10) and **ln** (base e ≈ 2.718).
- **Equations on the GDC:** systems of up to three linear equations and polynomial equations are solved with the GDC's solvers. Write the equations you entered, then the solutions.
- **HL:** laws of logarithms, $\log_a(xy) = \log_a x + \log_a y$ and friends, and rational exponents such as $8^{2/3} = 4$.

BY THE END YOU CAN

- Simplify expressions with integer exponents.
- Use logarithms with base 10 and e to solve exponential equations, including in context.
- Use the GDC to solve systems of linear equations and polynomial equations.
- Use the laws of logarithms. HL
- Simplify expressions with rational exponents. HL

SYLLABUS NOTE

From first exams in 2029, logarithms leave the SL syllabus. SL students sitting exams up to November 2028 still need the logarithm section of this note; the exponent and equation sections stay.

1 Laws of exponents

Law	Example
$a^m \times a^n = a^{m+n}$	$x^3 \times x^4 = x^7$
$a^m \div a^n = a^{m-n}$	$5^6 \div 5^2 = 5^4$
$(a^m)^n = a^{mn}$	$(2^3)^2 = 2^6 = 64$
$a^{-n} = 1/a^n$	$10^{-3} = 0.001$
$a^0 = 1$	$7^0 = 1$

WORKED EXAMPLE

Simplify $(2x^3)^2 \times x^{-4}$.

1. $(2x^3)^2 = 4x^6$ M1
2. **$4x^6 \times x^{-4} = 4x^2$** A1

WORKED EXAMPLE

Solve $3 \times 2^x = 48$.

1. $2^x = 16$ M1
2. **$16 = 2^4$, so $x = 4$** A1

COMMON MISTAKE

Writing $(2x^3)^2 = 2x^6$. The power applies to the 2 as well: $2^2 = 4$.

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2 Logarithms

- **Definition:** $\log_{10} 1000 = 3$ because $10^3 = 1000$. $\ln x = \log_e x$, so $\ln e^5 = 5$.

$$a^x = b \Leftrightarrow x = \log_a b$$ FORMULA BOOKLET

$a > 0, b > 0$

WORKED EXAMPLE

Solve $10^x = 250$ and $e^x = 7$.

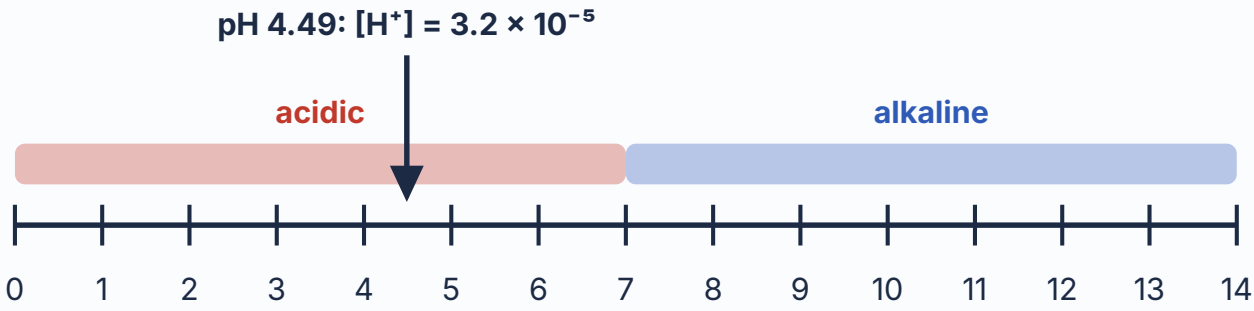
1. $x = \log_{10} 250 =$ **2.40** A1
2. **$x = \ln 7 = 1.95$** A1

WORKED EXAMPLE

A substance decays as $M = 80e^{-0.03t}$ grams after t years. When is the mass 20 g?

1. $e^{-0.03t} = 0.25$ M1
2. **$-0.03t = \ln 0.25 \Rightarrow t = 46.2$ years** M1 A1

- **Log scales:** $pH = -\log_{10}[H^+]$; sound level in decibels $= 10 \log_{10}(\frac{I}{I_0})$ with $I_0 = 10^{-12}$ W m^{-2}. A log scale squeezes huge ranges onto a short line.



$pH = -\log_{10}[H^+]$. One step on the scale is a factor of 10 in concentration.

Every step of 1 on the pH scale is a factor of 10 in hydrogen ion concentration.

WORKED EXAMPLE

Find the pH of a solution with $[H^+] = 3.2 \times 10^{-5}$ mol dm^{-3}, and the sound level of a noise with $I = 10^{-5}$ W m^{-2}.

1. pH $= -\log_{10}(3.2 \times 10^{-5}) =$ **4.49** A1
2. **$10 \log_{10}\left(\frac{10^{-5}}{10^{-12}}\right) = 10 \times 7 = 70$ dB** M1 A1

GDC (CALCULATOR) METHOD

To solve an exponential equation without logs, graph both sides ($y = 80e^{-0.03x}$ and $y = 20$) and find the intersection. This works on every paper in AI.

3 Solving equations with the GDC

WORKED EXAMPLE

Solve $2x + y - z = 2$, $x - y + 2z = 7$, $3x + 2y + z = 11$.

1. enter the coefficients into the simultaneous equation solver M1
2. **$x = 2$, $y = 1$, $z = 3$** A1

WORKED EXAMPLE

A shop sells 3 coffees and 2 cakes for \$19, and 1 coffee and 4 cakes for \$23. Find the price of each.

1. $3c + 2k = 19$ and $c + 4k = 23$ M1
2. **coffee \$3, cake \$5** A1

WORKED EXAMPLE

Solve $x^3 - 6x^2 + 9x - 2 = 0$.

1. polynomial root finder, degree 3, coefficients 1, −6, 9, −2 M1
2. **$x = 0.268, 2, 3.73$** A1

EXAM TIP

Always write the equations you solved before the GDC answer. "From GDC: x = 2" with no equations can lose the method mark if the answer is wrong.

4 Laws of logarithms HL

$$\log_a xy = \log_a x + \log_a y,\ \log_a \frac{x}{y} = \log_a x - \log_a y,\ \log_a x^m = m \log_a x$$ FORMULA BOOKLET

the three laws

$$\log_b x = \frac{\log_a x}{\log_a b}$$ FORMULA BOOKLET

change of base

WORKED EXAMPLE

Evaluate $\log 2 + \log 50$, and solve $\log_2 x + \log_2(x - 2) = 3$.

1. $\log(2 \times 50) = \log 100 = \mathbf{2}$ A1
2. $\log_2[x(x - 2)] = 3 \Rightarrow x^2 - 2x - 8 = 0$ M1
3. **$x = 4$ or -2; $x = 4$ ($x > 2$ for the logs to exist)** A1 R1

5 Rational exponents HL

- $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = (\sqrt[n]{a})^m$. So $8^{2/3} = (\sqrt[3]{8})^2 = 4$ and $27^{-1/3} = \frac{1}{3}$.

WORKED EXAMPLE

Simplify $\frac{x^{1/2} \times x^{3/2}}{x^{-1}}$.

1. $x^{1/2 + 3/2} = x^2$ M1
2. **$x^2 \div x^{-1} = x^3$** A1

LINKING QUESTION

Exponential equations are the backbone of Modelling with functions, and log scales are used to linearise data in Transformations and linearising data (HL). In Maths AA, logarithms are proved and used algebraically: see Exponents and logarithms.

IDEA FOR YOUR IA / EXPLORATION

Investigate a real logarithmic scale: earthquake magnitudes, decibels or pH. Collect data and show how the log scale makes comparisons easier, or model the cooling of a drink with an exponential function.

Test yourself — cover the answers first

QUICK CHECK

1. Simplify $x^5 \div x^{-2}$.
2. Evaluate 4^{-2}.
3. Solve $10^x = 0.01$.

4. What is ln e?
5. A solution has pH 3. What is $[H^+]$?
6. **HL** Write $\log 5 + \log 4$ as a single log.

EXAM-STYLE

7. The value of a painting is $V = 1200 \times 1.08^t$ dollars, t years after it was bought. Find the value after 5 years and how long it takes to reach $5000.
8. **HL** Solve $\log_3(x + 1) - \log_3 x = 2$.

Show answers

1. x^7
2. 1/16
3. $x = -2$
4. 1
5. 10^{-3} mol dm^{-3}
6. log 20
7. $V(5) = 1200 \times 1.08^5 =$ **$1763.19**
   $1.08^t = \frac{5000}{1200} \Rightarrow t = \frac{\ln(25/6)}{\ln 1.08}$ **= 18.5 years**
8. $\log_3 \frac{x+1}{x} = 2 \Rightarrow \frac{x+1}{x} = 9$
   $x + 1 = 9x$
   $x = \frac{1}{8}$

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