IB DP MATHS AI · TOPIC 5 CALCULUS · SL 5.5, 5.8 · AHL 5.11–5.13 · ~7 MIN READ

Integration, areas and kinematics

Integration as anti-differentiation, boundary conditions, definite integrals and areas with the GDC, the trapezoidal rule for data and functions, and for HL: integrating sin, cos, e^x and 1/x, integration by inspection, areas between curves and the y-axis, volumes of revolution, and kinematics with displacement and distance.

Anti-differentiation | + C and boundary conditions | Definite integrals | Area under a curve | Trapezoidal rule | Integrating sin, cos, e^x, 1/x (HL) | Integration by inspection (HL) | Area with the y-axis (HL)

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam
M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Integration reverses differentiation:** $\int ax^n\,dx = \frac{ax^{n+1}}{n+1} + C$ ($n \neq -1$). A **boundary condition** (a known point) fixes C.
- **Definite integrals** $\int_a^b f(x)\,dx$ give the area between the curve and the x-axis when $f(x) \geq 0$. In AI you evaluate them on the GDC.
- **Trapezoidal rule:** split the area into n strips of width h and add trapezia: $A \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + \ldots + y_{n-1})]$. It works on data tables too.
- **HL:** standard integrals, volumes of revolution $V = \pi\int y^2\,dx$, and kinematics: displacement = $\int v\,dt$, distance = $\int |v|\,dt$.

BY THE END YOU CAN

- Find indefinite integrals of ax^n and use boundary conditions to find C.
- Find areas under curves using definite integrals and the GDC.
- Use the trapezoidal rule to approximate areas from functions and data.
- Integrate sin x, cos x, e^x, 1/x and composites by inspection. HL
- Find areas between curves and the x- or y-axis, and volumes of revolution. HL
- Use calculus in kinematics: displacement, velocity, acceleration and distance. HL

SYLLABUS NOTE

From first exams in 2029, the trapezoidal rule leaves the AI SL syllabus. SL students sitting exams up to November 2028 still need it.

1 Anti-differentiation

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \; n \neq -1$$ FORMULA BOOKLET

add 1 to the power, divide by the new power

WORKED EXAMPLE

Find $\int(3x^2 - 4x + 5)\,dx$.

1. **$x^3 - 2x^2 + 5x + C$** A1 A1

WORKED EXAMPLE

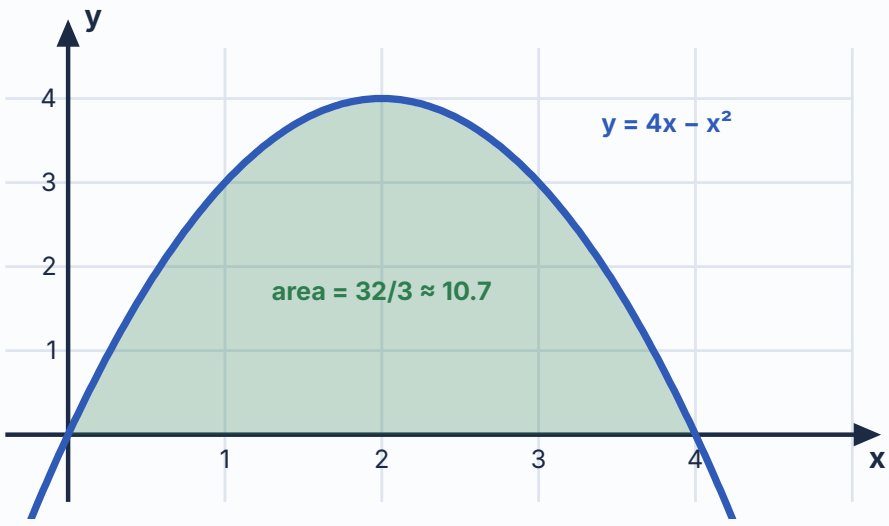
The gradient of a curve is $f'(x) = 6x - 2$, and the curve passes through $(1, 4)$. Find $f(x)$.

1. $f(x) = 3x^2 - 2x + C$ A1
2. $3 - 2 + C = 4 \Rightarrow C = 3$ M1
3. **$f(x) = 3x^2 - 2x + 3$** A1

COMMON MISTAKE

Forgetting + C on an indefinite integral. It costs a mark, and without it you can't use the boundary condition.

2 Definite integrals and area



The area between $y = 4x - x^2$ and the x-axis, from one root to the other.

WORKED EXAMPLE

Find the area enclosed by $y = 4x - x^2$ and the x-axis.

1. roots $x = 0$ and $x = 4$ **A1**
2. area $= \int_0^4 (4x - x^2)\,dx$ **M1**
3. $= \frac{32}{3} \approx 10.7$ (GDC) **A1**

- **Below the axis:** an integral is negative where the curve is below the x-axis. For a total area, use $\int|f(x)|\,dx$ on the GDC.

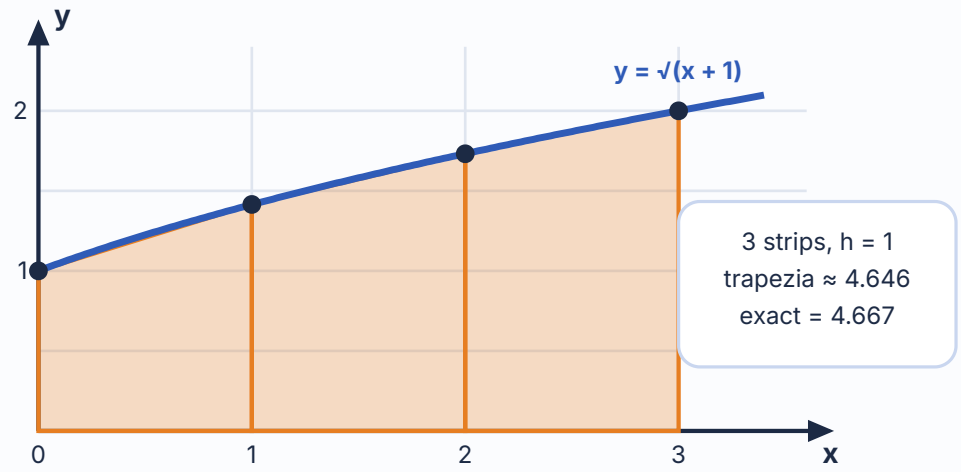
GDC (CALCULATOR) METHOD

Use the GDC's ∫f(x)dx (fnInt) tool. Write the integral with its limits in your working, then the value: "$\int_0^4 (4x - x^2)\,dx = 10.7$".

3 The trapezoidal rule

$$\int_a^b y\,dx \approx \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})],\ h = \frac{b-a}{n}$$ FORMULA BOOKLET

n strips of equal width h



Three trapezia under $y = \sqrt{(x + 1)}$. They sit just under the curve, so the estimate is a little too small.

WORKED EXAMPLE

Use the trapezoidal rule with 3 strips to estimate $\int_0^3 \sqrt{(x+1)}\,dx$, and find the percentage error.

1. h = 1; y-values 1, 1.414, 1.732, 2 A1
2. $\frac{1}{2}[(1+2)+2(1.414+1.732)] = \mathbf{4.65}$ (4.6463) M1 A1
3. **exact = 4.667 (GDC), so $\varepsilon = \frac{|4.6463-4.6667|}{4.6667} \times 100\% = 0.437\%$** M1 A1

WORKED EXAMPLE

A river is 4 m wide. Its depths at 1 m intervals across are 0, 1.2, 2.1, 1.8 and 0 m. Estimate the cross-sectional area.

1. h = 1 A1
2. $\frac{1}{2}[(0+0)+2(1.2+2.1+1.8)]$ M1
3. **= 5.1 m^2** A1

EXAM TIP

Over or under? If the curve is concave down (bulging upwards), the trapezia lie under it and underestimate the area; if it is concave up, they overestimate. Use more strips for a better estimate.

4 Standard integrals and integration by inspection HL

f(x)	∫ f(x) dx
x^n $(n \neq -1)$	$\frac{x^{n+1}}{n+1} + C$
$\frac{1}{x}$	$\ln x + C$ $(x > 0)$
$\sin x$	$-\cos x + C$
$\cos x$	$\sin x + C$
e^x	$e^x + C$
$\frac{1}{\cos^2 x}$	$\tan x + C$

- **By inspection (reverse chain rule):** $\int e^{3x}\,dx = \frac{1}{3}e^{3x} + C$; $\int \cos(2x)\,dx = \frac{1}{2}\sin(2x) + C$; $\int \frac{2x}{x^2+1}\,dx = \ln(x^2+1) + C$, because the top is the derivative of the bottom.

5 Areas and volumes HL

$V = \int_a^b \pi y^2\,dx$ (about the x-axis), $V = \int_a^b \pi x^2\,dy$ (about the y-axis) FORMULA BOOKLET

volume of revolution

WORKED EXAMPLE

The region under $y = x^2 + 1$ from $x = 0$ to $x = 2$ is rotated 360° about the x-axis. Find the volume.

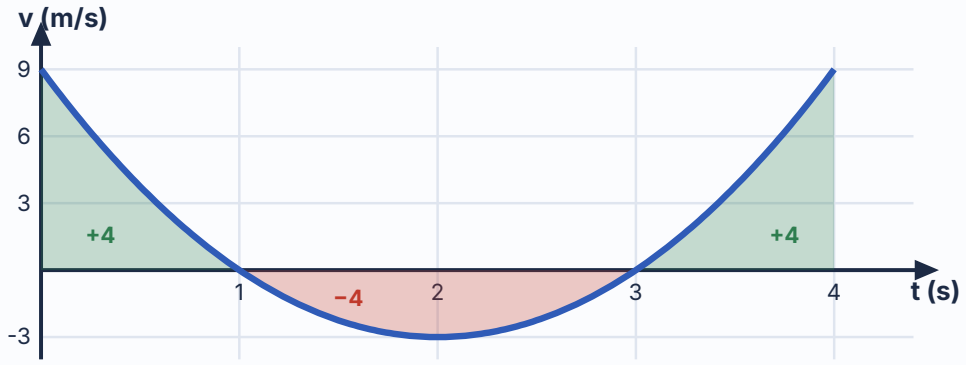
1. $V = \pi\int_0^2 (x^2 + 1)^2\,dx$ M1 A1
2. **$= 43.1$ (GDC; exactly $\frac{206\pi}{15}$)** A1

- **Area with the y-axis:** write x in terms of y and integrate with respect to y between y-limits: $A = \int_c^d x\,dy$.

6 Kinematics HL

$v = \frac{ds}{dt}$, $a = \frac{dv}{dt}$; displacement $= \int_{t_1}^{t_2} v\,dt$, distance $= \int_{t_1}^{t_2} |v|\,dt$ FORMULA BOOKLET

differentiate down, integrate up



$v = 3t^2 - 12t + 9$: displacement $= 4 + (-4) + 4 = 4$ m

distance $= 4 + 4 + 4 = 12$ m

Area above the t-axis is motion forward; below is motion backward. Distance counts both as positive.

WORKED EXAMPLE

A particle has velocity $v = 3t^2 - 12t + 9$ m s^{-1} for $0 \le t \le 4$. Find its displacement, the total distance travelled, and its acceleration at $t = 3$.

1. $\int_0^4 v\,dt$ = **4 m** M1 A1
2. $\int_0^4 |v|\,dt$ = **12 m** ($v = 0$ at $t = 1$ and 3) M1 A1
3. **$a = 6t - 12$, so $a(3) = 6$ m s^{-2}** A1

LINKING QUESTION

Integration reverses Differentiation and optimisation, and it is the method for solving Differential equations and phase portraits. The percentage error formula comes from Approximation, standard form and errors. For integration by hand in Maths AA, see Integration and kinematics.

IDEA FOR YOUR IA / EXPLORATION

Estimate the area of a real shape (a lake on a map, a leaf, a running track) with the trapezoidal rule, then improve it by fitting a function and integrating, and compare the two methods.

Test yourself — cover the answers first

QUICK CHECK

1. Find $\int 6x^2\,dx$.
2. Find $\int (2x + 1)\,dx$.
3. Evaluate $\int_1^3 2x\,dx$.
4. With $h = 0.5$ and 4 strips, what are a and b if $a = 0$?
5. Is the trapezoidal rule an over- or underestimate for a concave-down curve?
6. HL Find $\int e^{2x}\,dx$.

EXAM-STYLE

7. The rate of flow of water into a tank is $r(t) = 40 - 2t$ litres per minute for $0 \le t \le 20$. The tank starts with 50 litres. Find the volume after 10 minutes.
8. A car's speed (m s^{-1}) is recorded every 2 s: 0, 6, 10, 13, 15. Use the trapezoidal rule to estimate the distance travelled in 8 s.

Show answers

1. $2x^3 + C$
2. $x^2 + x + C$
3. 8
4. b = 2
5. an underestimate
6. $\frac{1}{2}e^{2x} + C$
7. $V = 50 + \int_0^{10} (40 - 2t)\,dt = 50 + [40t - t^2]_0^{10}$
   $= 50 + 300$ **= 350 litres**
8. h = 2
   $\frac{2}{2}[(0 + 15) + 2(6 + 10 + 13)] = 15 + 58$
   ≈ 73 m

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