IB DP MATHS AI · TOPIC 2 FUNCTIONS · SL 2.5, 2.6 · AHL 2.9 · ~8 MIN READ

Modelling with functions

The modelling cycle and the families of models IB Maths AI uses: linear and piecewise, quadratic, exponential growth and decay, direct and inverse variation, cubic and sinusoidal models, finding parameters from data, and for HL: logistic, natural logarithm and further piecewise models.

Modelling cycle | Linear & piecewise models | Quadratic models | Exponential growth & decay | Direct & inverse variation | Cubic models | Sinusoidal models | Fitting parameters

RS by Riyaz Sir
mentorriyaz.in

No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam
M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

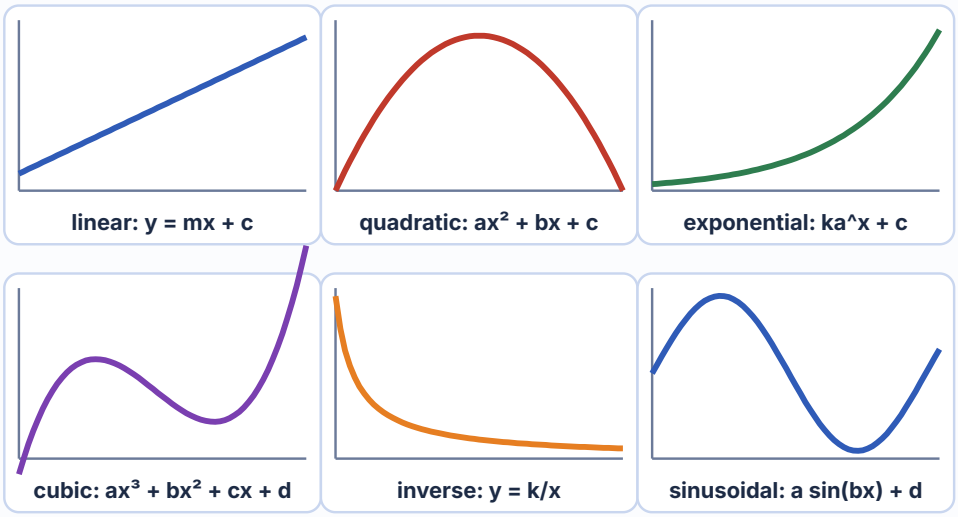
- **The modelling cycle:** choose a type of function from the shape of the data or the situation; find its parameters; use it to predict; then check whether the answers make sense and where the model breaks down.
- **Recognise the shape:** constant rate → linear; symmetric with one turning point → quadratic; constant percentage change → exponential; a repeating cycle → sinusoidal.
- **Exponential models** $f(x) = ka^x + c$ or $ke^{rx} + c$ have a horizontal asymptote $y = c$.
- **Sinusoidal models** $f(x) = a\sin(b(x - c)) + d$: amplitude $|a|$, period $\frac{360°}{b}$ (or $\frac{2\pi}{b}$ in radians, HL), principal axis $y = d$.
- **HL:** logistic models $\frac{L}{1 + Ce^{-kx}}$ level off at the carrying capacity L; logarithmic models $a + b \ln x$ grow ever more slowly.

BY THE END YOU CAN

- Choose an appropriate model from a description, a graph or data.
- Find the parameters of linear, quadratic, exponential, direct and inverse variation, cubic and sinusoidal models.
- Use a model to make predictions, and comment on its validity and limitations.
- Use logistic, logarithmic and piecewise models. HL

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1 Which model?



The six model families in the SL course. Learn their shapes so you can spot them in data.

Situation	Model
fixed fee + constant rate	linear $y = mx + c$
projectile, area, profit with one best price	quadratic $y = ax^2 + bx + c$
population, interest, radioactive decay, cooling	exponential $y = ka^x + c$
y doubles when x doubles	direct variation $y = kx^n$
y halves when x doubles	inverse variation $y = k/x$
tides, daylight, Ferris wheels, temperature over a year	sinusoidal $y = a \sin(bx) + d$

2 Quadratic and cubic models

WORKED EXAMPLE

A ball's height is $h = -4.9t^2 + 14t + 1.5$ m after t s. Find the maximum height and when the ball lands.

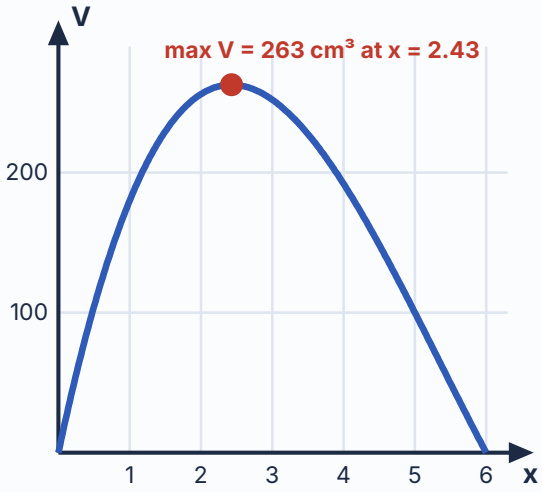
1. vertex at $t = \frac{-14}{2(-4.9)} = 1.43$ s **M1**
2. **$h_{max} = 11.5$ m** **A1**
3. **h = 0 at t = 2.96 s (GDC; the other root is negative, so reject it)** **A1** **R1**



cut x × x squares from each corner

$$V = x(20 - 2x)(12 - 2x)$$

domain $0 < x < 6$



An optimisation model: the GDC's maximum tool finds the best cut.

WORKED EXAMPLE

Squares of side x cm are cut from the corners of a 20 cm by 12 cm sheet, and the sides folded up to make an open box. Find the maximum volume.

1. $V = x(20 - 2x)(12 - 2x)$, $0 < x < 6$ M1 A1
2. GDC maximum: $x = 2.43$ cm A1
3. **$V_{max} = 263$ cm^3** A1

EXAM TIP

Always state the domain of a model from the context. Here $0 < x < 6$, because the 12 cm side can't lose more than 12 cm.

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3 Exponential models

$$f(x) = ka^x + c,\ f(x) = ke^{rx} + c$$

a (or e^r) is the growth factor; $y = c$ is the asymptote; $f(0) = k + c$

WORKED EXAMPLE

A bacteria colony starts with 500 cells and doubles every 3 hours. Write a model and find the number after 12 hours.

1. $N = 500 \times 2^{t/3}$ A1
2. **$N(12) = 500 \times 2^4 = 8000$ cells** A1

WORKED EXAMPLE

A population is 200 at t = 0 and 450 at t = 4. Model it as $P = 200e^{kt}$ and predict P at t = 10.

1. $200e^{4k} = 450 \Rightarrow k = \frac{\ln 2.25}{4} = 0.203$ M1 A1
2. **$P(10) = 200e^{2.03} \approx 1520$** A1

- **Cooling** follows $T = 20 + 70 \times 0.9^t$: the temperature drops towards the room temperature 20 °C, the horizontal asymptote, but never reaches it.

COMMON MISTAKE

Predicting far outside the data. An exponential population model predicts endless growth; real populations run out of food and space. Say this in "comment on the model" questions.

4 Direct and inverse variation

- **Direct:** $y \propto x^n$ means $y = kx^n$. **Inverse:** $y \propto \frac{1}{x^n}$ means $y = \frac{k}{x^n}$. Find k from one pair of values, then use it.

WORKED EXAMPLE

y is directly proportional to x^2, and y = 50 when x = 5. Find y when x = 8.

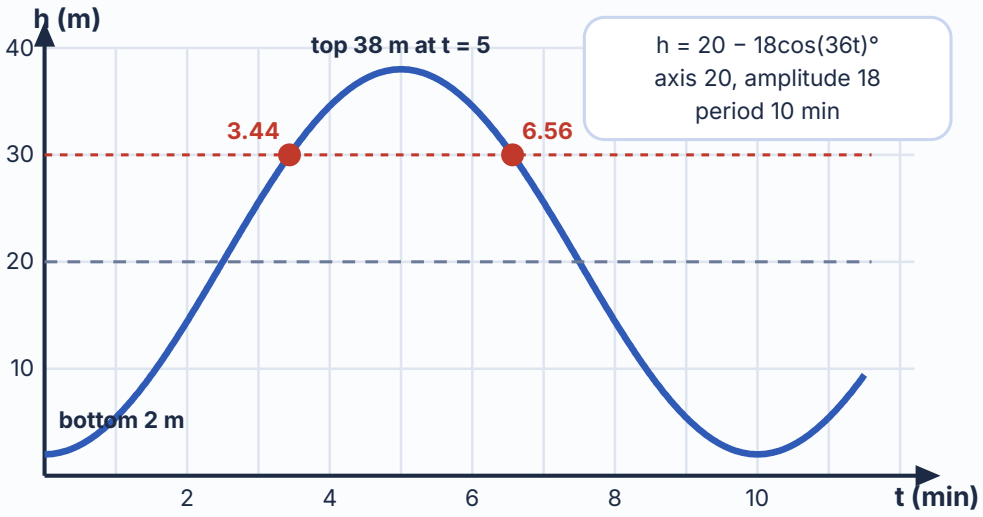
1. $50 = k \times 25 \Rightarrow k = 2$ M1 A1
2. **$y = 2 \times 64 = 128$** A1

WORKED EXAMPLE

Light intensity I is inversely proportional to the square of the distance d. I = 36 lux at 2 m. Find I at 3 m.

1. $36 = \frac{k}{4} \Rightarrow k = 144$ M1
2. **$I = \frac{144}{9} = 16$ lux** A1

5 Sinusoidal models



Read the axis, amplitude and period straight from the graph.

- **From a graph or data:** d = $\frac{\max + \min}{2}$, amplitude a = $\frac{\max - \min}{2}$, period = time from one peak to the next, b = $\frac{360°}{\text{period}}$ (or $\frac{2\pi}{\text{period}}$ in radians).

WORKED EXAMPLE

A Ferris wheel of diameter 36 m has its lowest point 2 m above the ground and turns once every 10 minutes. A rider starts at the bottom. Find a model for the height h and when the rider first reaches 30 m.

1. axis 20 m, amplitude 18 m, period 10 min **A1**
2. **h = 20 − 18cos(36t)°** (in radians, HL: h = 20 − 18cos($\frac{\pi t}{5}$)) **A1**
3. **GDC: h = 30 first at t = 3.44 min** **A1**

- **Why −cos?** cos starts at its maximum; −cos starts at its minimum, which matches a rider starting at the bottom.

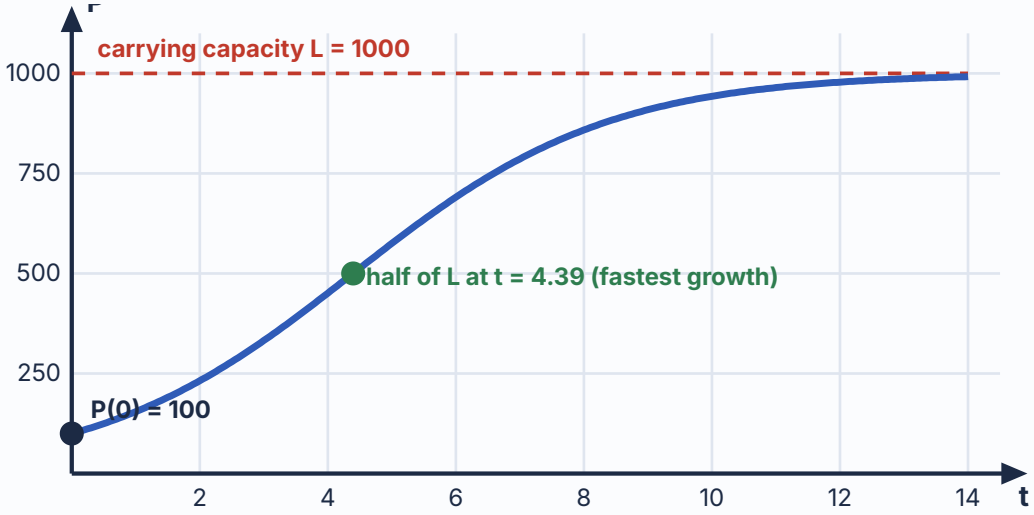
GDC (CALCULATOR) METHOD

To fit a model to data, use the regression tools: linear, quadratic, cubic, exponential, power and sinusoidal regression. Write the model with its parameters to 3 s.f., and give R^2 if asked how good the fit is.

6 Logistic and logarithmic models HL

$$f(x) = \frac{L}{1 + Ce^{-kx}},\ L, C, k > 0$$

logistic: L is the carrying capacity



P = 1000/(1 + 9e^(−0.5t)): fast growth at first, then levelling off

Logistic growth: exponential at first, then the curve levels off at the carrying capacity L.

WORKED EXAMPLE

A fish population is $P = \frac{1000}{1 + 9e^{-0.5t}}$, t in years. Find the initial population, the long-term population and when it reaches 500.

1. $P(0) = \frac{1000}{10} = 100$ A1
2. as $t \to \infty$, $e^{-0.5t} \to 0$, so $P \to 1000$ A1
3. $1 + 9e^{-0.5t} = 2 \Rightarrow t = \frac{\ln 9}{0.5} = 4.39$ years M1 A1

- **Logarithmic models** $f(x) = a + b \ln x$ $(x > 0)$ grow fast at first and then ever more slowly, without an asymptote: for example, perceived loudness against intensity.
- **Piecewise models** use different rules on different intervals: a phone plan, a tax system, or a car that accelerates and then cruises. Check that the pieces join where they should.

LINKING QUESTION

Exponential models connect to Sequences and financial mathematics (geometric growth), sinusoidal models need the trig of Trigonometry, 3D geometry and Voronoi diagrams, and HL students can check a model's type by linearising data (Composites, transformations and linearising data). Optimisation with calculus is in Differentiation and optimisation.

IDEA FOR YOUR IA / EXPLORATION

Collect real data (cooling of a drink, phone battery discharge, daylight hours, a bouncing ball from a video) and compare two or three models. Justify your choice with the context and R^2, and discuss where the model fails.

Test yourself — cover the answers first

QUICK CHECK

1. Which model fits "the value falls by 12% every year"?
2. State the asymptote of $y = 5 \times 0.7^x + 3$.
3. $y \propto 1/x$ and $y = 4$ when $x = 6$. Find y when $x = 8$.
4. Find the period of $y = 4 \sin(30x)° + 2$.
5. State the amplitude and axis of $y = 7 - 3\cos x$.
6. HL What is the carrying capacity of $P = 600/(1 + 5e^{-0.2t})$?

EXAM-STYLE

7. The temperature in a city is modelled by $T = 8\sin(30(t - 4))° + 18$, where t is the month (t = 1 is January). Find the maximum temperature and the month in which it happens.
8. A model for the value of a car is $V = 30\,000 \times 0.85^t + 2000$. Find the value after 6 years, interpret the 2000, and comment on using the model for t = 30.

Show answers

1. exponential, $y = k(0.88)^x$
2. $y = 3$
3. 3
4. 12
5. amplitude 3, axis $y = 7$
6. 600
7. max = 18 + 8 = **26 °C**
   $30(t - 4) = 90 \Rightarrow$ **t = 7, July**
8. $V(6) = 30\,000 \times 0.85^6 + 2000 =$ **$13 314.49**
   2000 is the lowest value the model allows (scrap value): the horizontal asymptote
   at t = 30 the model gives about $2228; a real 30-year-old car may be worth nothing, or more as a classic, so the prediction is unreliable

Stuck on a question?

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