IB DP MATHS AI · TOPIC 3 GEOMETRY AND TRIGONOMETRY · AHL 3.7–3.13 · ~8 MIN READ

Radians, vectors and matrix transformations

HL geometry for Maths AI: radians, arcs and sectors, the unit circle and Pythagorean identity, solving trig equations, geometric transformations with 2 × 2 matrices, vectors and the scalar product, vector equations of lines, vector kinematics with closest approach, and the vector product.

Radians | Arc length & sector area | Unit circle | $\sin^2\theta + \cos^2\theta = 1$ | Trig equations | Matrix transformations | Composite transformations | Determinant as area factor

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam
M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Radians:** π rad = 180°. With θ in radians, arc length $l = r\theta$ and sector area $A = \frac{1}{2}r^2\theta$.
- **Unit circle:** the point at angle θ is (cos θ, sin θ), so $\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \frac{\sin\theta}{\cos\theta}$.
- **Matrix transformations:** a 2 × 2 matrix maps (x; y) to a new point. $|\det|$ is the **area scale factor**. For "A then B" the matrix is **BA**.
- **Vectors:** $\mathbf{v}\cdot\mathbf{w} = |\mathbf{v}||\mathbf{w}|\cos\theta$ gives angles; $\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ is a line; $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$ describes constant-velocity motion, with speed $|\mathbf{v}|$.
- This note is HL only. The **vector product** section leaves the course in 2029 (see the Syllabus note).

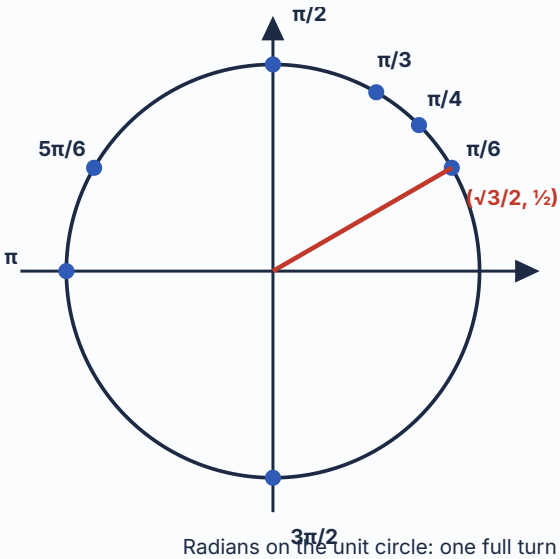
BY THE END YOU CAN

- Use radians, including arc length and sector area. HL
- Use the unit circle, $\sin^2\theta + \cos^2\theta = 1$ and $\tan\theta = \sin\theta/\cos\theta$, and solve trig equations in a given interval. HL
- Describe and apply transformations with matrices, including composites, and use the determinant as an area scale factor. HL
- Use vectors, the scalar product and the angle between vectors. HL
- Write vector equations of lines and use them in kinematics, including closest approach. HL
- Find and use the vector product. HL

SYLLABUS NOTE

From first exams in 2029, the vector (cross) product leaves the AI HL syllabus. Everything else here stays. If you sit exams up to November 2028, you still need the vector product too.

1 Radians and the unit circle HL



π rad = 180°
x = cos θ
y = sin θ
$\sin^2\theta + \cos^2\theta = 1$

Radians on the unit circle: one full turn is 2π.

Learn the key angles in radians and their coordinates on the unit circle.

$l = r\theta$, $A = \frac{1}{2}r^2\theta$ FORMULA BOOKLET

θ in radians

WORKED EXAMPLE

Convert 150° to radians, and find the arc length of a sector of radius 8 cm and angle 1.2 rad.

1. $150 \times \frac{\pi}{180} = \frac{5\pi}{6}$ A1
2. **$l = 8 \times 1.2 = 9.6$ cm** A1

$\cos^2\theta + \sin^2\theta = 1$, $\tan\theta = \frac{\sin\theta}{\cos\theta}$ FORMULA BOOKLET

identities

WORKED EXAMPLE

θ is obtuse and sin θ = 0.6. Find cos θ and tan θ.

1. $\cos^2\theta = 1 - 0.36 = 0.64$, and cos θ < 0 for obtuse θ M1 R1
2. **cos θ = −0.8, tan θ = −0.75** A1

WORKED EXAMPLE

Solve 2 sin x = 1 for 0 ≤ x ≤ 2π.

1. sin x = 0.5 M1
2. $x = \frac{\pi}{6}$ and $\pi - \frac{\pi}{6}$ M1
3. **$x = \frac{\pi}{6}, \frac{5\pi}{6}$ (or 0.524, 2.62)** A1

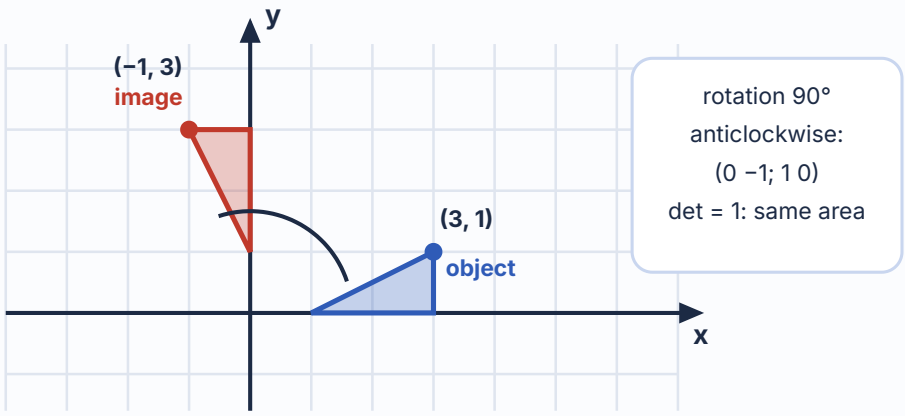
GDC (CALCULATOR) METHOD

Set the GDC to radians, graph $y = 2\sin x$ and $y = 1$ on $0 \le x \le 2\pi$, and use intersect. Check the window matches the interval exactly.

2 Matrix transformations HL

Transformation	Matrix
rotation θ anticlockwise about O	$\begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}$
reflection in $y = (\tan\theta)x$	$\begin{pmatrix}\cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$
enlargement, scale factor k, centre O	$\begin{pmatrix}k & 0 \\ 0 & k\end{pmatrix}$
horizontal stretch, factor k	$\begin{pmatrix}k & 0 \\ 0 & 1\end{pmatrix}$
vertical stretch, factor k	$\begin{pmatrix}1 & 0 \\ 0 & k\end{pmatrix}$

- The rotation and reflection matrices are in the formula booklet. For any other transformation, see where (1; 0) and (0; 1) go: those images are the two columns of the matrix. So the rotation by 90° is $\begin{pmatrix}0 & -1 \\ 1 & 0\end{pmatrix}$, sending (1; 0) to (0; 1).



The rotation matrix moves each vertex; the shape keeps its area because det = 1.

WORKED EXAMPLE

The triangle with vertices (1, 0), (3, 0), (3, 1) is rotated 90° anticlockwise about the origin. Find the image of (3, 1).

1. $\begin{pmatrix}0 & -1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}3 \\ 1\end{pmatrix}$ M1
2. **(−1; 3)** A1

- **Composite:** "transformation A followed by B" has matrix **BA** (the first one goes next to the vector).
- **Area:** area of image = $|\det M|$ × area of object. A negative determinant means the shape has been reflected.

WORKED EXAMPLE

A shape of area 4 cm^2 is transformed by M = (3 1; 1 2). Find the area of the image.

1. det M = 6 − 1 = 5 **A1**
2. **area = 5 × 4 = 20 cm^2** **A1**

COMMON MISTAKE

Multiplying in the wrong order for composites. "Reflect, then rotate" is (rotation)(reflection), not the other way round.

3 Vectors and the scalar product HL

$$|\mathbf{v}| = \sqrt{(v_1^2 + v_2^2 + v_3^2)}$$ FORMULA BOOKLET

magnitude

$$\mathbf{v} \cdot \mathbf{w} = v_1w_1 + v_2w_2 + v_3w_3 = |\mathbf{v}||\mathbf{w}| \cos\theta$$ FORMULA BOOKLET

scalar product; 0 means perpendicular

WORKED EXAMPLE

Find the angle between a = (3, 0, 4) and b = (2, 2, 1).

1. a · b = 6 + 0 + 4 = 10, |a| = 5, |b| = 3 **A1**
2. $\cos\theta = \frac{10}{15}$ **M1**
3. **θ = 48.2°** **A1**

- **Position vectors:** OA is the vector from the origin to A, and AB = b − a.

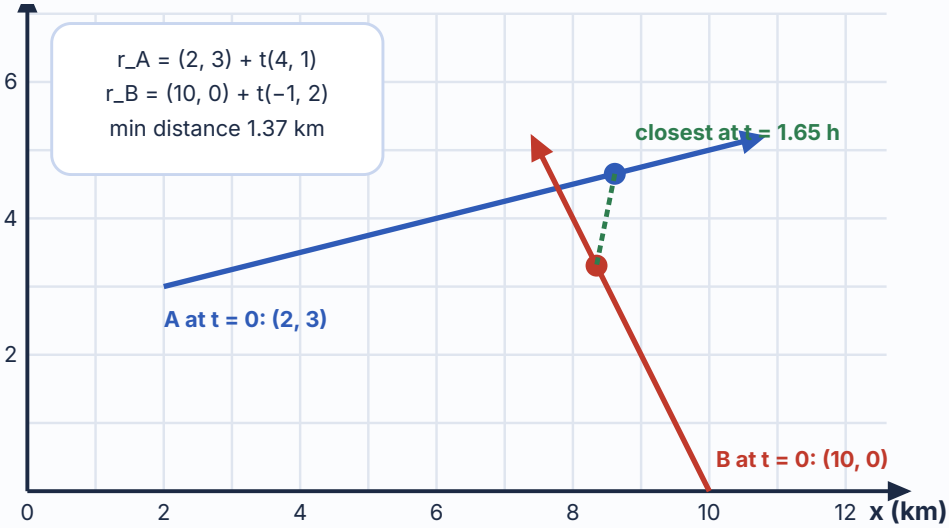
4 Vector equations and kinematics HL

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$$ FORMULA BOOKLET

a line through a with direction b

$$\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$$

position at time t; v is the velocity, |v| the speed



Two ships on straight courses. Their distance apart is smallest when the line joining them is perpendicular to their relative velocity.

WORKED EXAMPLE

Ship A is at (2, 3) + t(4, 1) km and ship B at (10, 0) + t(−1, 2) km, t in hours. Find the speed of A and the closest distance between the ships.

1. speed of A = $|(4, 1)| = \sqrt{17} =$ **4.12 km h^{-1}** A1
2. B − A = (8 − 5t, −3 + t) M1
3. $d^2 = (8 - 5t)^2 + (t - 3)^2$; minimum (GDC or calculus) at $t = \frac{43}{26} = 1.65$ h M1 A1
4. **closest distance 1.37 km** A1

- **Do they collide?** Only if they are at the same point at the **same** time t. Paths that cross at different times are not a collision.

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5 The vector product HL

$$\mathbf{v} \times \mathbf{w} = (v_2w_3 - v_3w_2, v_3w_1 - v_1w_3, v_1w_2 - v_2w_1), \quad |\mathbf{v} \times \mathbf{w}| = |\mathbf{v}||\mathbf{w}| \sin\theta$$ FORMULA BOOKLET

perpendicular to both v and w

WORKED EXAMPLE

Find (1, 2, 0) × (0, 1, 3), and the area of the parallelogram these vectors make.

1. (2 × 3 − 0 × 1, 0 × 0 − 1 × 3, 1 × 1 − 2 × 0) M1
2. **= (6, −3, 1)** A1
3. **area = √(36 + 9 + 1) = √46 ≈ 6.78** A1

LINKING QUESTION

Radians and sinusoidal models connect to Modelling with functions; rotation matrices and complex numbers both rotate the plane (Complex numbers); velocity vectors return in Differential equations and phase portraits. For lines and planes in 3D with more algebra, see Maths AA Vectors, lines and planes.

IDEA FOR YOUR IA / EXPLORATION

Model the flight paths of two aircraft or drones with vector equations and find their closest approach, or use matrix transformations to explain how a phone app rotates and resizes photos.

Test yourself — cover the answers first

QUICK CHECK

1. HL Convert 2 rad to degrees.
2. HL Find the sector area for r = 4 and θ = 0.5 rad.
3. HL Write the matrix for an enlargement with scale factor 3 about O.
4. HL det M = −2. What happens to the area and orientation?
5. HL Find |(2, −3, 6)|.
6. HL What does v · w = 0 mean?

EXAM-STYLE

7. HL Find the matrix of a reflection in y = x followed by a rotation of 90° anticlockwise about O, and describe the single transformation.
8. HL A particle moves with r = (1, 4) + t(3, −2). Find when it crosses the x-axis and its speed.

Show answers

1. 115°
2. 4
3. (3 0; 0 3)
4. area doubles; the shape is reflected
5. 7
6. v and w are perpendicular
7. reflection (0 1; 1 0), rotation (0 −1; 1 0)
   combined = (0 −1; 1 0)(0 1; 1 0) = **(−1 0; 0 1)**
   a reflection in the y-axis
8. y = 4 − 2t = 0 ⇒ **t = 2** (at (7, 0))
   speed = √(9 + 4) **= √13 ≈ 3.61**

Stuck on a question?

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