IB DP MATHS AI · TOPIC 1 NUMBER AND ALGEBRA · SL 1.2–1.4, 1.7 · AHL 1.11 · ~8 MIN READ

Sequences and financial mathematics

Arithmetic and geometric sequences and series in context, sigma notation, compound interest, depreciation and inflation, and loans, amortisation and annuities with the GDC finance (TVM) solver, plus the sum to infinity for HL.

Arithmetic sequences & series | Geometric sequences & series | Sigma notation | Compound interest | Depreciation | Inflation & real value | Loans & amortisation | Annuities

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No tag = **SL and HL** HL ONLY HL students only FORMULA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

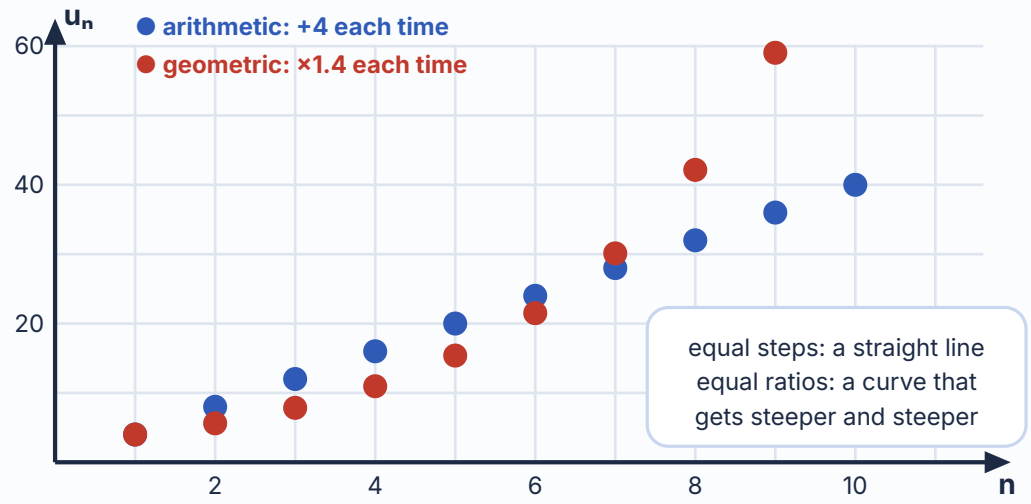
IN 30 SECONDS

- **Arithmetic:** add the same d each time. $u_n = u_1 + (n - 1)d$ and $S_n = \frac{n}{2}(2u_1 + (n - 1)d)$. Use it for linear growth: seats in rows, simple interest.
- **Geometric:** multiply by the same r each time. $u_n = u_1 r^{n-1}$ and $S_n = \frac{u_1(r^n - 1)}{r - 1}$. Use it for percentage growth and decay.
- **Compound interest:** $FV = PV \times (1 + \frac{r}{100k})^{kn}$, where k is the number of compounding periods per year and n the number of years.
- **Loans and annuities:** use the GDC finance solver (TVM). Money you receive is **positive**, money you pay out is **negative**.
- **HL:** a geometric series with $|r| < 1$ has a sum to infinity $S_\infty = \frac{u_1}{1 - r}$.

BY THE END YOU CAN

- Use arithmetic and geometric sequences and series to model real situations.
- Use sigma notation.
- Calculate compound interest, depreciation and the effect of inflation.
- Use the GDC finance solver for loans, amortisation and annuities, and interpret the results.
- Find and use the sum to infinity of a geometric series. HL

1 Arithmetic and geometric sequences



Arithmetic sequences grow by equal steps; geometric sequences grow by equal ratios.

$u_n = u_1 + (n-1)d$, $S_n = \frac{n}{2}(2u_1 + (n-1)d) = \frac{n}{2}(u_1 + u_n)$ FORMULA BOOKLET

arithmetic

$u_n = u_1 r^{n-1}$, $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$, $r \neq 1$

FORMULA BOOKLET

geometric

WORKED EXAMPLE

A theatre has 20 seats in the first row and 2 more seats in each row after that. There are 25 rows. Find the number of seats in the last row and in the theatre.

1. $u_{25} = 20 + 24 \times 2 =$ **68 seats** M1 A1
2. $S_{25} = \frac{25}{2}(2 \times 20 + 24 \times 2) =$ **1100 seats** M1 A1

WORKED EXAMPLE

In an arithmetic sequence $u_3 = 11$ and $u_8 = 31$. Find u_1 and d.

1. $u_8 - u_3 = 5d = 20$, so **d = 4** M1 A1
2. **$u_1 = 11 - 2 \times 4 = 3$** A1

WORKED EXAMPLE

A town of 2500 people grows by 4% a year. Find the population after 10 years, and the first year in which it is more than double.

1. r = 1.04: $2500 \times 1.04^{10} = 3700.6$, so **about 3700 people** M1 A1
2. $1.04^n > 2 \Rightarrow n > 17.7$ (GDC table or solver) M1
3. **after 18 years** A1

COMMON MISTAKE

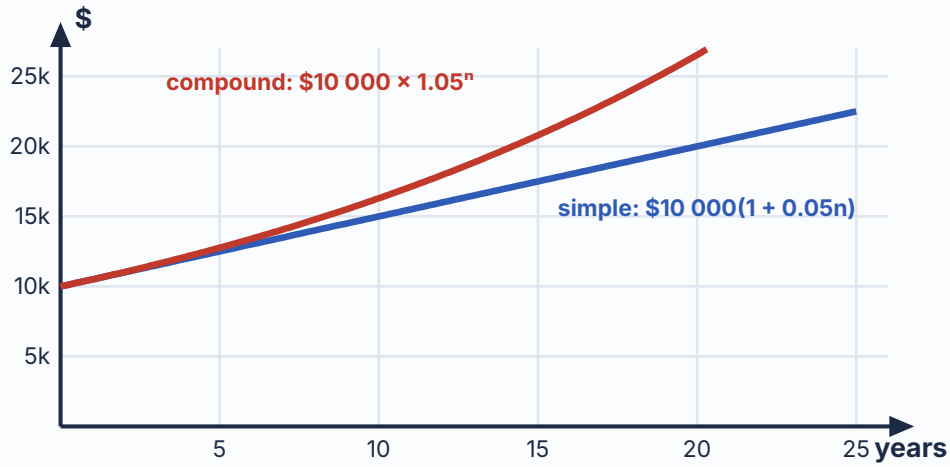
Using r = 0.04 for a 4% increase. The multiplier is 1 + 0.04 = **1.04**; for a 4% decrease it is 0.96.

- **Sigma notation:** $\Sigma_{k=1}^{10}(3k + 2)$ means add 3k + 2 for k = 1, 2, ..., 10. It is arithmetic with $u_1 = 5$, $u_{10} = 32$, so the sum is $\frac{10}{2}(5 + 32) = 185$.

GDC (CALCULATOR) METHOD

The sequence and sum functions, or a table of values, are the fastest way to find "the first n for which...". Write down the two values either side of the boundary as your evidence.

2 Compound interest, depreciation and inflation



Compound interest earns interest on the interest, so it grows geometrically.

Over 25 years, compound interest leaves simple interest far behind.

$$FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$$ FORMULA BOOKLET

k = compounding periods per year, n = years

WORKED EXAMPLE

$5000 is invested at 3.5% per year, compounded quarterly, for 8 years. Find the final value.

1. $FV = 5000\left(1 + \frac{3.5}{400}\right)^{32}$ M1 A1
2. **= $6607.60** A1

WORKED EXAMPLE

A car worth $25 000 depreciates by 15% each year. Find its value after 4 years.

1. $25\,000 \times 0.85^4$ M1
2. **= $13 050.16** A1

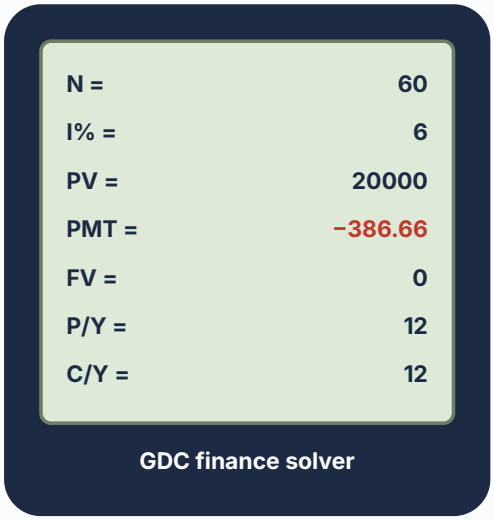
- **Inflation** reduces what money can buy. A quick IB method: real interest rate ≈ interest rate − inflation rate. $10 000 at 5% with 2% inflation for 10 years has a real value of about $10\,000 \times 1.03^{10}$ = $13 439 in today's money.

EXAM TIP

Money answers: give them to 2 decimal places (cents), or to the nearest whole unit if the question says so. "Correct to 3 s.f." does not apply to money.

3 Loans, amortisation and annuities (GDC)

loan of $20 000 at 6% p.a., monthly, 5 years



PV > 0: money you receive
PMT < 0: money you pay out

The finance solver screen. Fill in every value you know, set the unknown to 0, then solve for it.

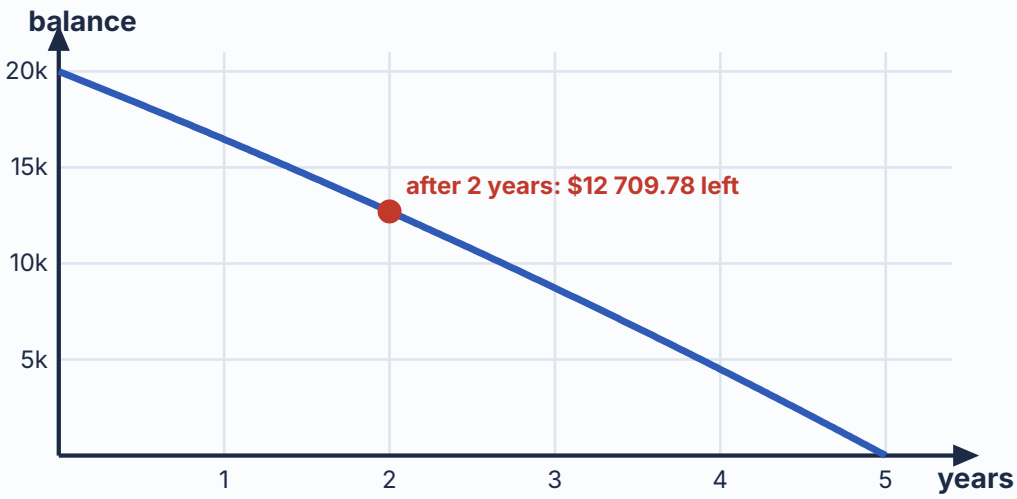
Symbol	Meaning
N	total number of payments (years × payments per year)
I%	annual interest rate, as a percentage
PV	present value: + for a loan you receive, − for money you invest
PMT	each regular payment: − if you pay it
FV	future value: 0 when a loan is paid off
P/Y, C/Y	payments and compounding periods per year

WORKED EXAMPLE

$20 000 is borrowed at 6% per year, compounded monthly, and repaid with equal monthly payments over 5 years. Find the monthly payment, the total paid and the total interest.

1. N = 60, I% = 6, PV = 20 000, FV = 0, P/Y = C/Y = 12 M1
2. PMT = **−$386.66 a month** A1
3. **total paid = 60 × 386.66 = $23 199.36, so the interest is $3199.36** M1 A1

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Early payments are mostly interest, so the balance falls slowly at first.

The amortisation curve: the balance owed at the end of each month.

WORKED EXAMPLE

For the same loan, find the amount still owed after 2 years.

1. N = 24, PMT = −386.656..., solve for FV **M1**
2. **FV = −12 709.78, so $12 709.78 is still owed** **A1** **R1**

WORKED EXAMPLE

Ali saves $200 at the end of every month in an account paying 4% per year compounded monthly. Find the amount after 10 years and the interest earned.

1. N = 120, I% = 4, PV = 0, PMT = −200, P/Y = C/Y = 12 **M1**
2. FV = **$29 449.96** **A1**
3. **interest = 29 449.96 − 24 000 = $5449.96** **A1**

COMMON MISTAKE

Getting the signs wrong in the TVM solver. If PV and PMT have the same sign for a loan, the GDC gives an error or nonsense. Think about the direction the money moves.

- **Write the setup, not just the answer.** Examiners give method marks for the TVM values. Copy them into your answer as a list: N = 60, I% = 6, PV = 20 000, ...

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4 Sum to infinity HL

$$S_\infty = \frac{u_1}{1 - r}, \ |r| < 1$$ FORMULA BOOKLET

the series converges only when $-1 < r < 1$

WORKED EXAMPLE

A ball is dropped from 5 m and each bounce reaches 60% of the previous height. Find the total distance it travels before it stops.

1. down: $5 + 3 + 1.8 + \ldots = \frac{5}{1 - 0.6} = 12.5$ m M1 A1
2. up: $3 + 1.8 + \ldots = \frac{3}{0.4} = 7.5$ m M1
3. **total = 20 m** A1

LINKING QUESTION

Geometric sequences are discrete versions of exponential models (Modelling with functions), and the same multiplier idea gives exponential growth and decay with the number e (Exponents, logarithms and equations). In Maths AA the algebra of series goes further: see Sequences, series and the binomial theorem.

IDEA FOR YOUR IA / EXPLORATION

Compare real loan offers (car loans, mortgages or student loans) with the TVM solver, and investigate how the term and the rate change the total interest. Or model a real population or savings plan with a geometric sequence and test the model against data.

Test yourself — cover the answers first

QUICK CHECK

1. Find u_{20} for 7, 11, 15, ...
2. Find the multiplier for a 12% decrease.
3. Write $\Sigma_{k=1}^{4} 2^k$ as a sum and evaluate it.
4. In the TVM solver, what is FV for a loan that is fully repaid?
5. What does I% = 6 with C/Y = 12 mean?
6. HL Does $4 + 6 + 9 + \ldots$ have a sum to infinity?

EXAM-STYLE

7. A salary starts at $40 000 and rises by 3% each year. Find the salary in year 10 and the total earned over the first 10 years.
8. How many whole years does it take for $10 000 to reach $15 000 at 5% per year, compounded monthly?

Show answers

1. 83
2. 0.88
3. 2 + 4 + 8 + 16 = 30
4. 0
5. 6% per year, compounded monthly
6. no, r = 1.5 > 1
7. $u_{10} = 40\,000 \times 1.03^9 =$ **$52 190.93**

   $S_{10} = \frac{40\,000(1.03^{10} - 1)}{0.03} =$ **$458 555.17**
8. $10\,000(1 + \frac{5}{1200})^{12n} = 15\,000$

   n = 8.13 years (GDC)

   so **9 years** (it has not reached $15 000 after 8 years)

Stuck on a question?

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