IB DP PHYSICS · THEME A SPACE, TIME AND MOTION · A.4 (HL ONLY) · ~9 MIN READ

A.4 Rigid body mechanics

Torque and rotational equilibrium, angular displacement, velocity and acceleration, the rotational suvat equations, moment of inertia, Newton's second law for rotation, angular momentum and rotational kinetic energy.

Torque | Rotational equilibrium | Angular kinematics | Moment of inertia | τ = Iα | Angular momentum | Rotational kinetic energy

RS by Riyaz Sir
mentorriyaz.in

No tag = **SL and HL** HL ONLY HL students only DATA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **A.4 is HL only.** It is the rotational version of A.1–A.3: every linear quantity has a rotational partner (s → θ, v → ω, a → α, m → I, F → τ, p → L).
- **Torque τ = Fr sin θ** turns a body. Zero resultant torque AND zero resultant force means equilibrium.
- **Moment of inertia I = Σmr²** measures how hard a body is to spin up. It depends on where the mass is, not just how much.
- **τ = Iα**, **L = Iω** (conserved when no resultant torque acts) and **E_k = ½Iω²** complete the set.

BY THE END YOU CAN

- Calculate torques and solve rotational equilibrium problems. HL
- Use angular displacement, velocity and acceleration, and the equations for uniform angular acceleration. HL
- Calculate moments of inertia for point masses and use given values for other shapes. HL
- Apply τ = Iα, angular impulse, conservation of angular momentum and rotational kinetic energy. HL

1 Torque and equilibrium HL

Only the component F sin θ, at 90° to r, turns the bar.



τ = Fr sin θ
unit: N m

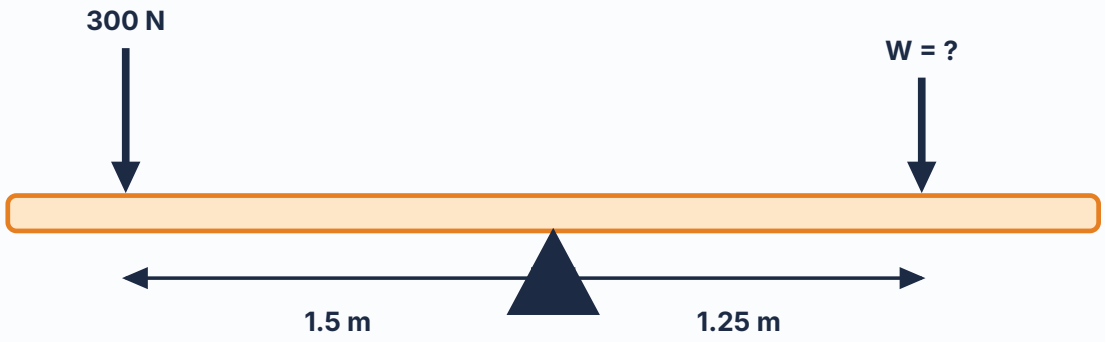
A force pushing straight along the bar towards the pivot cannot turn it. Only the perpendicular component counts.

$$\tau = Fr \sin \theta$$ DATA BOOKLET

θ is the angle between F and r; τ in N m

- **Same unit as energy, different idea:** write torque in N m, never in J.
- **Couple:** two equal, opposite, parallel forces. Zero resultant force but a non-zero torque: pure rotation.

Rotational equilibrium — the resultant torque about ANY axis is zero. Full equilibrium also needs the resultant force to be zero (translational equilibrium).



Clockwise moments = anticlockwise moments: 300 × 1.5 = W × 1.25, so W = 360 N

Equilibrium also needs zero resultant force: the pivot pushes up with 660 N.

Take moments about the pivot: the unknown pivot force then drops out of the torque equation.

WORKED EXAMPLE

A uniform 4.0 m plank of weight 200 N rests on two supports, one at each end. A 600 N person stands 1.0 m from the left end. Find the force from each support.

1. moments about the left support: R × 4.0 = 600 × 1.0 + 200 × 2.0 M1
2. R = 1000/4.0 = **250 N** (right support) A1
3. **vertical forces: L + 250 = 800, so L = 550 N** A1

EXAM TIP

Choose the axis through the force you know least about. Its torque is zero there, so it disappears from the equation.

Riyaz Sir

2 Describing rotation HL

Angular displacement θ — the angle turned through, in radians.

Angular velocity ω — rate of change of angle, $\omega = \Delta\theta/\Delta t$, in rad s^{-1}. Linked to linear speed by $v = \omega r$.

Angular acceleration α — rate of change of ω, in rad s^{-2}. Linked to the tangential acceleration by $a = \alpha r$.

Linear	Rotational (uniform α) DATA BOOKLET
$v = u + at$	$\omega_f = \omega_i + \alpha t$
$s = ut + \frac{1}{2}at^2$	$\Delta\theta = \omega_i t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$
$s = \frac{1}{2}(u + v)t$	$\Delta\theta = \frac{1}{2}(\omega_f + \omega_i)t$

WORKED EXAMPLE

A wheel spinning at 30 rad s^{-1} slows uniformly to rest in 12 s. Find α and the number of turns.

1. $\alpha = (0 - 30)/12 =$ **-2.5 rad s^{-2}** A1
2. $\Delta\theta = \frac{1}{2}(30 + 0)(12) = 180$ rad M1
3. **turns $= 180/2\pi = 28.6$** A1

COMMON MISTAKE

Using revolutions or degrees in the equations. Convert to radians first: 1 rev = 2π rad, and 600 rpm = $600 \times 2\pi/60 = 62.8$ rad s^{-1}.

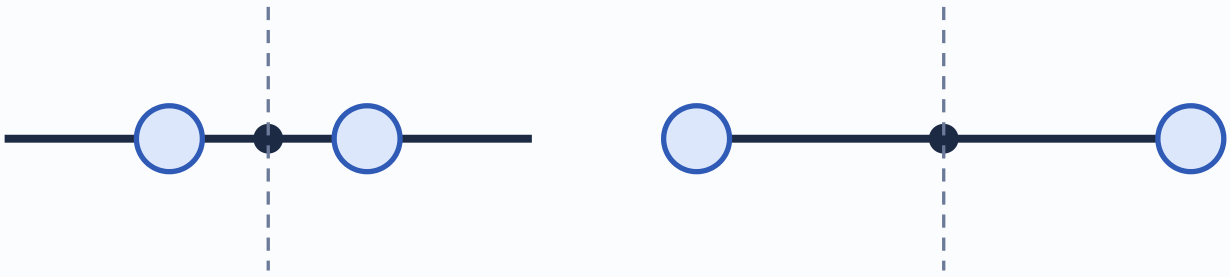
3 Moment of inertia and Newton's second law for rotation HL

$$I = \Sigma mr^2$$ DATA BOOKLET

r is the distance of each mass from the AXIS; I in kg m^2

Riyaz Sir

Same mass, same axis: the distribution decides $I = \Sigma mr^2$



masses near the axis
$I = 2m(0.10)^2$ small
easy to spin up

masses far out
$I = 2m(0.25)^2$ large
hard to spin up

Moving the same masses further from the axis raises I sharply, because r is squared.

- **Other shapes:** the exam gives the formula when needed, for example a solid cylinder about its axis $I = \frac{1}{2}MR^2$, a hoop $I = MR^2$, a solid sphere $I = \frac{2}{5}MR^2$.
- **I depends on the axis.** The same rod is easier to spin about its centre than about one end.

$$\tau = I\alpha$$ DATA BOOKLET

the resultant torque gives the angular acceleration

WORKED EXAMPLE

Two 0.50 kg masses sit at the ends of a light rod, each 0.40 m from the axis. A torque of 2.0 N m acts for 5.0 s from rest. Find I, ω and the angle turned.

1. $I = 2 \times 0.50 \times 0.40^2 =$ **$0.16\ \text{kg m}^2$** A1
2. $\alpha = \tau/I = 2.0/0.16 = 12.5\ \text{rad s}^{-2}$ M1
3. $\omega = 0 + 12.5 \times 5.0 =$ **$62.5\ \text{rad s}^{-1}$** A1
4. **$\Delta\theta = \frac{1}{2} \times 12.5 \times 5.0^2 = 156\ \text{rad}$** A1

WORKED EXAMPLE

A string wound round a pulley ($I = 0.020\ \text{kg m}^2$, radius 0.10 m) is pulled with a constant 5.0 N force. Find α and the linear acceleration of the string.

1. $\tau = Fr = 5.0 \times 0.10 = 0.50\ \text{N m}$ M1
2. $\alpha = 0.50/0.020 =$ **$25\ \text{rad s}^{-2}$** A1
3. **$a = \alpha r = 25 \times 0.10 = 2.5\ \text{m s}^{-2}$** A1

4 Angular momentum HL

$$L = I\omega$$ DATA BOOKLET

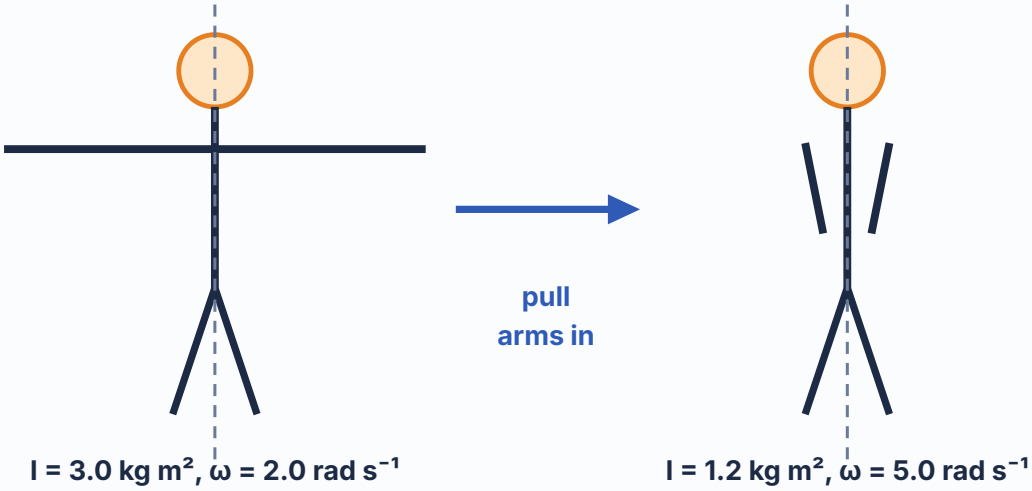
in kg m^2 s^{-1}

$$\Delta L = \tau \Delta t = \Delta(I\omega)$$ DATA BOOKLET

angular impulse equals the change in angular momentum

- **Conservation:** with no resultant external torque, total angular momentum stays constant. Divers, skaters, collapsing stars and spinning chairs all show it.

L = Iω is conserved: smaller I, faster spin



Pulling her arms in cuts I, so ω rises to keep Iω constant.

WORKED EXAMPLE

In the diagram, find the final angular velocity and the change in kinetic energy.

1. L conserved: 3.0 × 2.0 = 1.2ω M1
2. ω = **5.0 rad s^{-1}** A1
3. E_k: ½(3.0)(2.0²) = 6.0 J → ½(1.2)(5.0²) = 15 J M1
4. **E_k increases by 9.0 J: her muscles do work pulling her arms in** R1

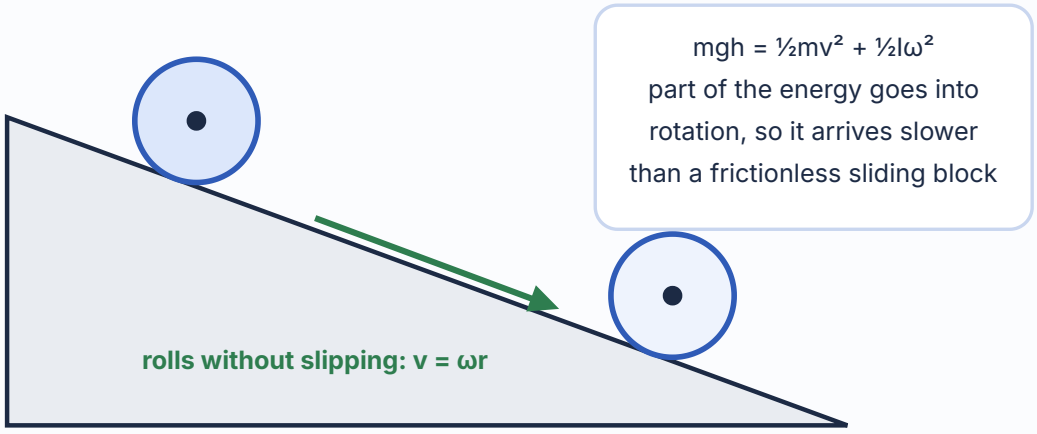
COMMON MISTAKE

Assuming rotational kinetic energy is conserved when I changes. Angular momentum is conserved; kinetic energy usually is not.

5 Rotational kinetic energy HL

$$E_k = \tfrac{1}{2}I\omega^2 = \frac{L^2}{2I}$$ DATA BOOKLET

energy stored in rotation



A rolling object shares its energy between moving along and spinning.

WORKED EXAMPLE

A solid cylinder ($I = \tfrac{1}{2}MR^2$) rolls without slipping from rest down a slope of vertical height 1.0 m. Find its speed at the bottom.

1. $Mgh = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}(\tfrac{1}{2}MR^2)(v/R)^2$ M1 M1
2. $gh = \tfrac{3}{4}v^2$ A1
3. **$v = \sqrt{(4gh/3)} = \sqrt{(4 \times 9.8 \times 1.0/3)} = 3.6\ \text{m s}^{-1}$ (a sliding block would reach $4.4\ \text{m s}^{-1}$)** A1

EXAM TIP

Flywheels store energy as $\tfrac{1}{2}I\omega^2$. A heavy rim (large I) at high ω stores the most, which is why flywheels are rim-heavy.

LINKING QUESTION

Every result in this topic mirrors one from A.1–A.3. Can you write the rotational version of "impulse equals change in momentum" and "work equals force × distance" ($W = \tau\Delta\theta$)?

IDEA FOR YOUR IA / EXPLORATION

Roll cylinders with the same mass and radius but different mass distributions (solid, hollow) down a ramp. Time them and deduce I from the energy equation.

Test yourself — cover the answers first

QUICK CHECK

1. What is the unit of torque?
2. State the two conditions for a rigid body to be in equilibrium.
3. Convert 300 rpm to rad s^{-1}.
4. Why does a hoop have a larger I than a disc of the same mass and radius?
5. When is angular momentum conserved?
6. A skater halves her moment of inertia. What happens to ω and E_k?

EXAM-STYLE

7. A door 0.90 m wide is pushed at its edge with 40 N at 60° to the plane of the door. Find the torque about the hinges.
8. A flywheel (I = 2.5 kg m^2) spins at 120 rad s^{-1}. A brake applies a torque of 15 N m. How long does it take to stop, and how much energy is dissipated?
9. A 40 kg child runs at 3.0 m s^{-1} along a tangent and jumps onto the edge of a stationary roundabout (I = 500 kg m^2, radius 2.0 m). Find the angular velocity afterwards.

Show answers

1. N m (not J)
2. zero resultant force and zero resultant torque
3. 300 × 2π/60 = 31.4 rad s^{-1}
4. all its mass is at the maximum distance from the axis
5. when no resultant external torque acts
6. ω doubles; E_k doubles
7. τ = Fr sin θ = 40 × 0.90 × sin 60°
   τ = 31 N m
8. α = −15/2.5 = −6.0 rad s^{-2}
   t = 120/6.0 = **20 s**
   energy = ½ × 2.5 × 120^2 = **1.8×10^4 J**
9. L before = mvr = 40 × 3.0 × 2.0 = 240 kg m^2 s^{-1}
   I after = 500 + 40 × 2.0^2 = 660 kg m^2
   ω = 240/660 = **0.36 rad s^{-1}**

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Share with a friend

Ask Riyaz Sir

Want regular help? Send this to your parent →