IB DP PHYSICS · THEME A SPACE, TIME AND MOTION · A.5 (HL ONLY) · ~8 MIN READ

A.5 Galilean and special relativity

Reference frames and Galilean transformations, Einstein's two postulates, the Lorentz transformations, time dilation, length contraction, simultaneity, the spacetime interval, spacetime diagrams and the muon experiment.

Reference frames | Galilean transformation | Postulates | Lorentz transformation | Time dilation | Length contraction | Spacetime diagrams | Muon decay

RS by Riyaz Sir
mentorriyaz.in

No tag = **SL and HL** HL ONLY HL students only DATA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

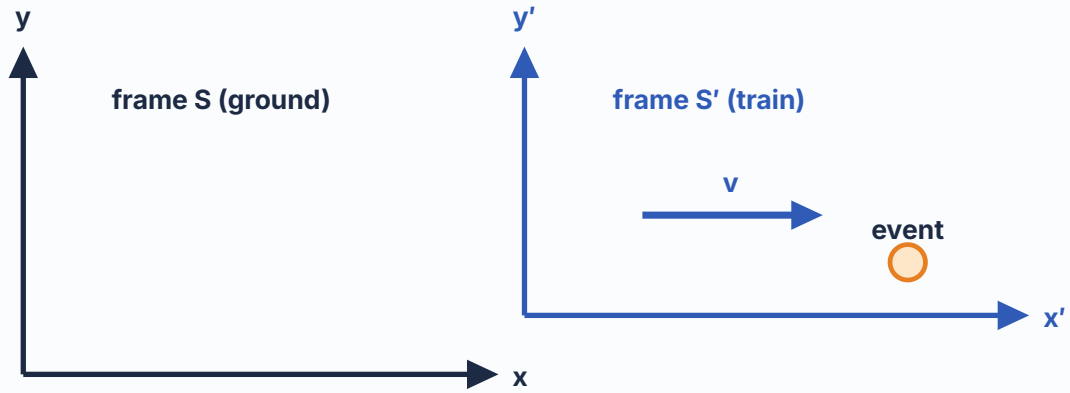
IN 30 SECONDS

- **A.5 is HL only.** A **reference frame** is a set of rulers and clocks. Newton's laws hold in every inertial (non-accelerating) frame.
- **Galilean relativity** adds velocities simply, $u' = u - v$. It fails near the speed of light, because **light has the same speed c in every inertial frame** (Einstein's second postulate).
- The **Lorentz transformations** replace the Galilean ones. They lead to **time dilation** $\Delta t = \gamma \Delta t_0$, **length contraction** $L = L_0/\gamma$ and the **relativity of simultaneity**.
- Observers disagree about times and lengths, but they all agree on the **spacetime interval** $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$.

BY THE END YOU CAN

- Apply the Galilean transformations and explain why they fail at high speed. HL
- State the two postulates of special relativity. HL
- Use the Lorentz transformations, γ and relativistic velocity addition. HL
- Identify proper time and proper length, and calculate time dilation and length contraction. HL
- Use the invariant spacetime interval and draw and read spacetime diagrams. HL
- Explain the muon experiment from both frames. HL

1 Reference frames and Galilean relativity HL



Galilean: x′ = x − vt, t′ = t, u′ = u − v. Fine at low speed, wrong near c.

S′ moves at speed v along the x-axis of S. At t = 0 the origins coincide.

Inertial frame — a frame moving at constant velocity, where an object with no resultant force keeps a constant velocity.

$$x' = x - vt,\ t' = t,\ u' = u - v$$ DATA BOOKLET

Galilean transformation and velocity addition

WORKED EXAMPLE

A train moves at 30 m s^{-1}. A passenger walks forwards at 2.0 m s^{-1} relative to the train. Find her speed relative to the ground and the speed of a light beam from the train's headlight, according to Galileo.

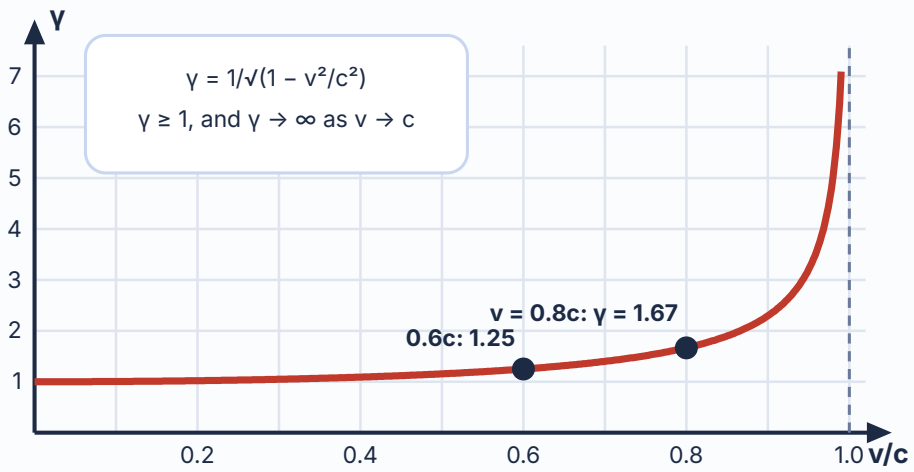
1. u = u′ + v = 2.0 + 30 = **32 m s^{-1}** ✓ A1
2. **light: c + 30 m s^{-1}, which contradicts experiment and Maxwell's equations** R1

2 Einstein's postulates HL

- **Postulate 1:** the laws of physics are the same in all inertial frames.
- **Postulate 2:** the speed of light in a vacuum is the same, c, for all inertial observers, whatever the motion of the source.

$$\gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}$$ DATA BOOKLET

the Lorentz factor



γ is almost exactly 1 at everyday speeds and shoots up near c.

3 Lorentz transformations HL

$$x' = \gamma(x - vt),\ t' = \gamma\left(t - \frac{vx}{c^2}\right)$$ DATA BOOKLET

coordinates of an event in S′

$$u' = \frac{u - v}{1 - uv/c^2}$$ DATA BOOKLET

relativistic velocity addition; never gives more than c

WORKED EXAMPLE

In S, an event happens at x = 100 m, t = 0.50 μs. Find its coordinates in S′ moving at 0.60c.

1. $\gamma = 1/\sqrt{(1 - 0.36)} =$ **1.25** A1
2. $x' = 1.25(100 - 0.60 \times 3.0 \times 10^8 \times 0.50 \times 10^{-6}) = 1.25(100 - 90) =$ **12.5 m** M1 A1
3. **$t' = 1.25(0.50 \times 10^{-6} - 0.60 \times 100/(3.0 \times 10^8)) = 0.375$ μs** A1

WORKED EXAMPLE

Two spaceships approach the Earth from opposite sides, each at 0.60c. Find the speed of one relative to the other.

1. Earth frame S: ship B has u = −0.60c; ship A is frame S′, moving at v = +0.60c M1
2. $u' = (u - v)/(1 - uv/c^2) = (-0.60c - 0.60c)/(1 + 0.36) = -0.88c$ A1
3. **speed 0.88c, not 1.2c** A1

4 Time dilation and length contraction HL

Proper time Δt_0 — the time between two events measured by a clock present at both, so the events happen at the same place in its frame. It is the shortest time any observer measures.

Proper length L_0 — the length of an object measured in its own rest frame. It is the longest length any observer measures.

$$\Delta t = \gamma \Delta t_0$$ DATA BOOKLET

moving clocks run slow

$$L = \frac{L_0}{\gamma}$$ DATA BOOKLET

moving objects are shorter along their direction of motion only

WORKED EXAMPLE

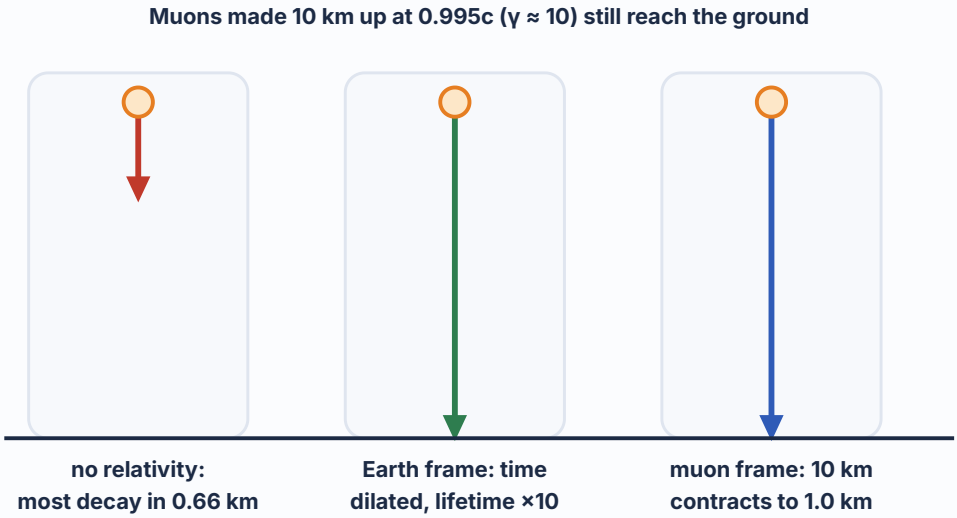
A spaceship travels at 0.80c to a star 12 light-years away (Earth frame). Find the journey time in the Earth frame and for the crew.

1. Earth: t = 12 ly ÷ 0.80c = **15 years** A1
2. γ = 1/0.60 = 1.67
3. crew clock is present at departure and arrival, so it shows proper time: 15/1.67 = **9.0 years** M1 A1
4. **for the crew the distance is contracted to 12/1.67 = 7.2 ly, and 7.2/0.80 = 9.0 years ✓** R1

COMMON MISTAKE

Swapping proper and dilated time. Ask: which observer has ONE clock present at both events? That clock measures the proper time, and it is the smaller value.

The muon experiment



Two frames, one result: the muons reach the ground.

- Muons have a mean lifetime of about 2.2 μs in their own frame. At 0.995c (γ ≈ 10) they would travel only 0.66 km on average without relativity, yet many reach the ground from 10 km up.
- **Earth frame:** their lifetime is dilated to about 22 μs, enough to travel about 6.6 km on average, so a measurable fraction survive 10 km. **Muon frame:** the lifetime is normal, but the atmosphere is contracted to about 1 km.

5 Simultaneity and the spacetime interval HL

- **Relativity of simultaneity:** events that are simultaneous in one frame are not simultaneous in a frame moving relative to it (unless they happen at the same place). This is the vx/c^2 term in the Lorentz transformation for t′.

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$$ DATA BOOKLET

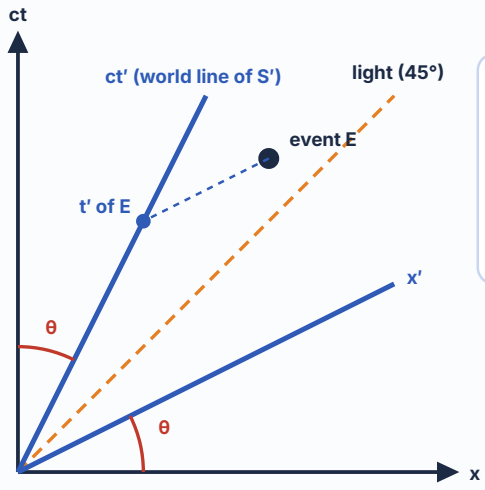
invariant: every inertial observer gets the same value

WORKED EXAMPLE

Two events are 600 m apart and 3.0 μs apart in the lab. Find the proper time between them.

1. $c\Delta t = 3.0 \times 10^8 \times 3.0 \times 10^{-6} = 900$ m M1
2. $(\Delta s)^2 = 900^2 - 600^2 = 4.5 \times 10^5$ m^2 M1
3. **in the frame where $\Delta x = 0$: $c\Delta t_0 = 671$ m, so $\Delta t_0 = 2.2$ μs** A1

6 Spacetime diagrams HL



tan θ = v/c
here v = 0.5c
both S′ axes tilt
by θ towards
the light line

The S′ axes are tilted towards the light line. A faster frame means a bigger tilt.

$$\tan\theta = \frac{v}{c}$$ DATA BOOKLET

θ is the angle between the world line (ct′ axis) and the ct axis

- **Reading coordinates in S′:** draw lines from the event parallel to the S′ axes. Lines parallel to the x′ axis are lines of equal t′, which is how the diagram shows simultaneity.
- **World lines** of objects slower than light are steeper than 45°; light travels along 45° lines.

EXAM TIP

In "explain using a spacetime diagram" questions, draw lines of simultaneity (parallel to x′) through the two events and show that they meet the ct′ axis at different points.

LINKING QUESTION

The muon experiment ties A.5 to E.3 (decay and lifetimes). Particle accelerators are designed with these same equations.

Test yourself — cover the answers first

QUICK CHECK

1. State Einstein's two postulates.
2. Calculate γ at 0.60c.
3. Which observer measures proper time?
4. A rod moves along its length at 0.80c. Its proper length is 3.0 m. What length is measured?
5. Is the spacetime interval between two events the same for all inertial observers?
6. On a spacetime diagram, what angle does a light ray make with the axes?

EXAM-STYLE

7. A particle with proper lifetime 26 ns moves at 0.90c in the lab. Find its lifetime and the distance it travels in the lab.
8. A rocket moving at 0.50c fires a probe forwards at 0.80c relative to the rocket. Find the probe's speed relative to Earth.

Show answers

1. Physics is the same in all inertial frames; c is the same for all inertial observers.
2. 1.25
3. the one for whom both events happen at the same place
4. 3.0/1.67 = 1.8 m
5. yes, it is invariant
6. 45°
7. γ = 1/√(1 − 0.81) = 2.29
   lab lifetime = 2.29 × 26 = **60 ns**
   distance = $0.90 \times 3.0 \times 10^{8} \times 60 \times 10^{-9}$ = **16 m**
8. $u = (u' + v)/(1 + u'v/c^2)$
   = 1.30c/(1 + 0.40)
   = **0.93c**

Stuck on a question?

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Ask Riyaz Sir