IB DP PHYSICS · THEME C WAVE BEHAVIOUR · C.1 · ~6 MIN READ

C.1 Simple harmonic motion

The conditions for SHM and a = −ω²x, period, frequency and angular frequency, the mass–spring system and the simple pendulum, energy changes, and (HL) phase angle, the x, v and energy equations.

Conditions for SHM | a = −ω²x | Period & angular frequency | Mass–spring | Pendulum | Energy in SHM | Phase angle (HL)

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No tag = **SL and HL** HL ONLY HL students only DATA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

IN 30 SECONDS

- **Simple harmonic motion** happens when the acceleration is proportional to the displacement from equilibrium and always directed back towards it: **a = −ω²x**.
- The period does not depend on the amplitude. For a spring **T = 2π√(m/k)**; for a pendulum (small swings) **T = 2π√(l/g)**.
- Energy swaps between kinetic and potential twice per cycle; the total stays constant (no damping).
- **HL:** $x = x_0 \sin(\omega t + \varphi)$, $v = \pm\omega\sqrt{(x_0^2 - x^2)}$ and $E_T = \frac{1}{2}m\omega^2 x_0^2$.

BY THE END YOU CAN

- State the conditions for SHM and explain the minus sign in $a = -\omega^2 x$.
- Use T, f and ω, and the mass–spring and pendulum period equations.
- Describe energy changes during an oscillation.
- Use phase angle and the displacement and velocity equations. HL
- Calculate kinetic, potential and total energy in SHM. HL

1 What makes motion simple harmonic

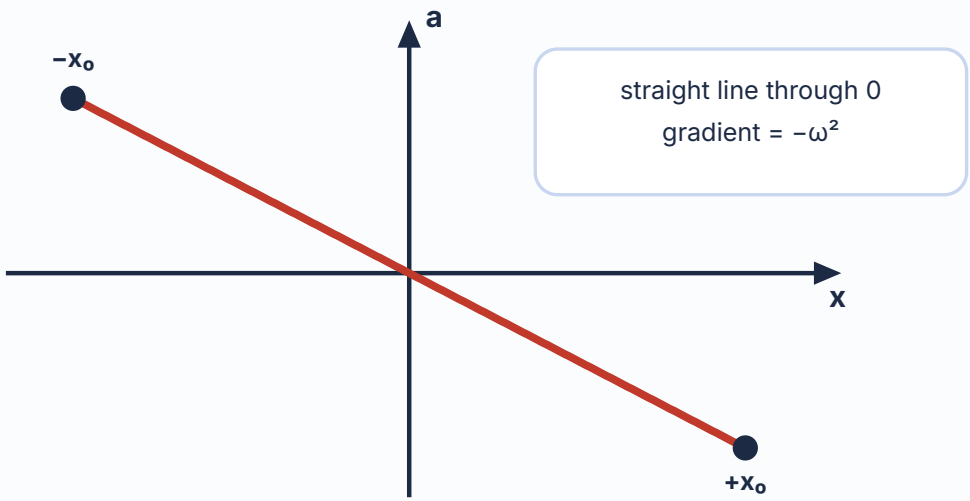
Equilibrium position — where the resultant force on the oscillator is zero.

Displacement x — distance from equilibrium, with a sign. Amplitude x_0 is the maximum displacement.

- **Conditions for SHM:** (1) a restoring force (and so acceleration) proportional to the displacement; (2) always directed towards the equilibrium position.

$$a = -\omega^2 x$$ DATA BOOKLET

ω = angular frequency in rad s^{-1}; the minus sign means a is opposite to x



The minus sign: acceleration always points back towards equilibrium.

The defining graph of SHM: a straight line through the origin with negative gradient.

$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$ DATA BOOKLET

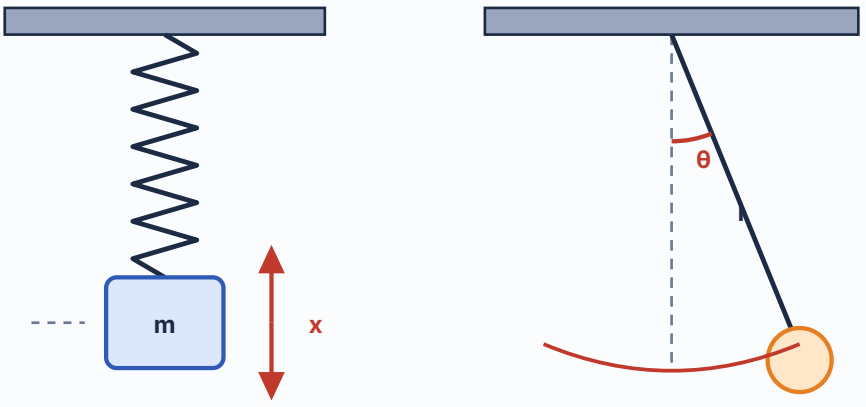
period T in s, frequency f in Hz

WORKED EXAMPLE

An oscillator has frequency 2.0 Hz and amplitude 5.0 cm. Find its maximum acceleration.

1. $\omega = 2\pi f = 12.6$ rad s^{-1} M1
2. $a_{max} = \omega^2 x_0 = 12.6^2 \times 0.050$ M1
3. **= 7.9 m s^{-2}, at the ends of the motion** A1

2 Mass–spring and pendulum



mass–spring
$T = 2\pi\sqrt{(m/k)}$

simple pendulum (small θ)
$T = 2\pi\sqrt{(l/g)}$

Two classic oscillators. Neither period depends on the amplitude (for small pendulum swings).

$$T = 2\pi\sqrt{\frac{m}{k}}$$ DATA BOOKLET

mass on a spring

$$T = 2\pi\sqrt{\frac{l}{g}}$$ DATA BOOKLET

simple pendulum, small angles (below about 10°)

WORKED EXAMPLE

A 0.40 kg mass hangs on a spring with k = 25 N m^{-1}. Find the period.

1. $T = 2\pi\sqrt{(0.40/25)}$ M1
2. **= 0.79 s** A1

WORKED EXAMPLE

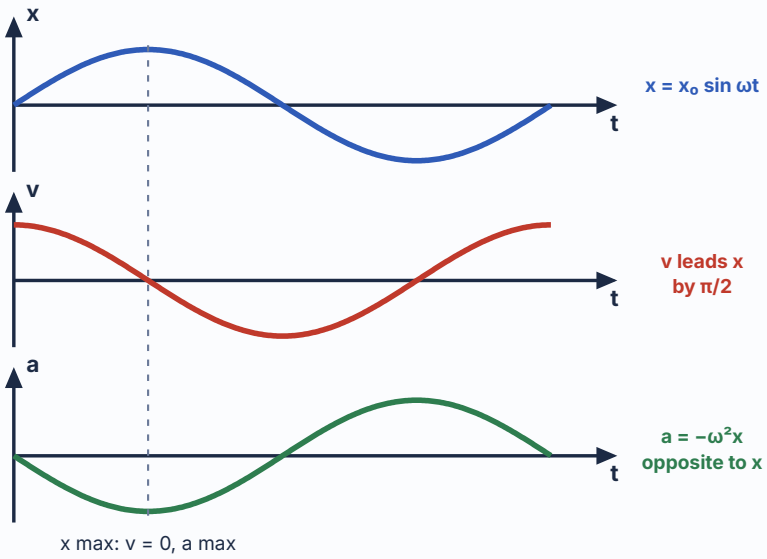
What length of pendulum has a period of 2.0 s?

1. $l = gT^2/4\pi^2 = 9.8 \times 4.0/39.5$ M1
2. **= 0.99 m** A1

COMMON MISTAKE

Thinking a heavier pendulum bob swings faster or slower. Mass cancels: the pendulum period depends only on l and g. (For a spring, mass does matter.)

3 Graphs of SHM



Where displacement is greatest, velocity is zero and acceleration is greatest (and opposite).

- **v is the gradient of the x–t graph** and **a is the gradient of the v–t graph**. v is a quarter of a cycle (π/2) ahead of x; a is half a cycle (π) out of phase with x.
- **Energy (qualitative):** at the ends, all potential energy ($E_k = 0$); at equilibrium, all kinetic energy (maximum speed). With damping, the total slowly falls (C.4).

4 Phase and the SHM equations HL

Phase angle φ — tells you where in the cycle the oscillator starts. Two oscillators are in phase if φ is the same; in antiphase if they differ by π.

$$x = x_0 \sin(\omega t + \varphi)$$ DATA BOOKLET

displacement (φ = 0 starts at equilibrium moving in +x; φ = π/2 starts at $+x_0$)

$$v = \omega x_0 \cos(\omega t + \varphi)$$ DATA BOOKLET

velocity; maximum speed $v_{max} = \omega x_0$

$$v = \pm\omega\sqrt{(x_0^2 - x^2)}$$ DATA BOOKLET

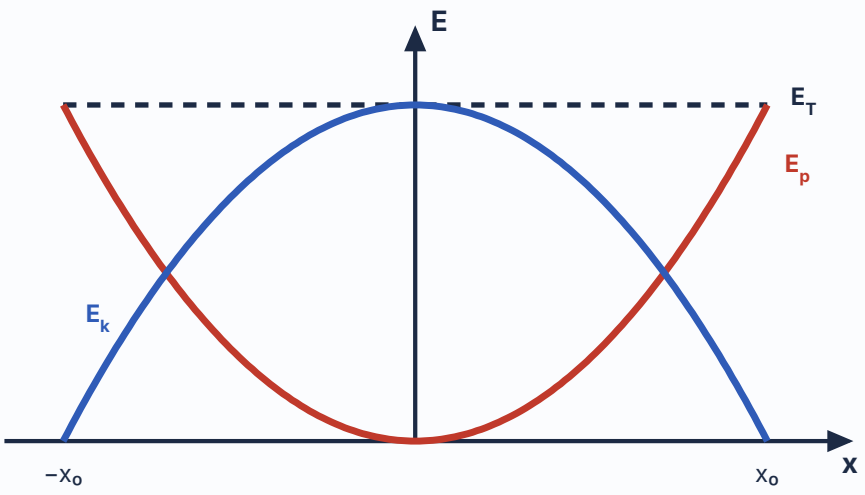
speed at any displacement

WORKED EXAMPLE

For the 2.0 Hz, 5.0 cm oscillator, find the maximum speed and the speed at x = 3.0 cm.

1. $v_{max} = \omega x_0 = 12.6 \times 0.050 =$ **$0.63\ m\ s^{-1}$** A1
2. $v = \omega\sqrt{(0.050^2 - 0.030^2)} = 12.6 \times 0.040$ M1
3. **$= 0.50\ m\ s^{-1}$** A1

5 Energy in SHM HL



$E_p = ½m\omega^2x^2$, $E_k = ½m\omega^2(x_o^2 - x^2)$: they add to $E_T = ½m\omega^2x_o^2$

Two parabolas that always add to the same flat total.

$$E_T = \tfrac{1}{2}m\omega^2 x_0^2$$ DATA BOOKLET

total energy, proportional to amplitude squared

$$E_p = \tfrac{1}{2}m\omega^2 x^2$$ DATA BOOKLET

potential energy at displacement x

WORKED EXAMPLE

A 0.20 kg mass oscillates at 2.0 Hz with amplitude 5.0 cm. Find the total energy and the kinetic energy at x = 3.0 cm.

1. $E_T = \tfrac{1}{2} \times 0.20 \times 12.6^2 \times 0.050^2 =$ **0.039 J** M1 A1
2. $E_p = \tfrac{1}{2} \times 0.20 \times 12.6^2 \times 0.030^2 = 0.014$ J
3. **$E_k = 0.039 - 0.014 = 0.025$ J** A1

EXAM TIP

Doubling the amplitude doubles v_{max} but quadruples the total energy ($E \propto x_0^2$). The period is unchanged.

LINKING QUESTION

SHM is the source of every wave in C.2: each particle of a medium oscillates in SHM as the wave passes. Orbits seen edge-on (D.1) also look like SHM.

IDEA FOR YOUR IA / EXPLORATION

Measure how the period of a pendulum depends on length, plot T^2 against l and find g from the gradient ($4\pi^2/g$). Extend it to large angles to see where SHM breaks down.

Test yourself — cover the answers first

QUICK CHECK

1. State the two conditions for SHM.
2. Where is the speed greatest in SHM?
3. A pendulum's length is quadrupled. What happens to its period?
4. A spring oscillator's mass is quadrupled. What happens to T?
5. What is the gradient of an a–x graph for SHM?
6. HL An oscillator starts at its maximum positive displacement. What is φ in $x = x_0 \sin(\omega t + \varphi)$?

EXAM-STYLE

7. A pendulum has a period of 2.0 s on Earth. What would its period be on the Moon ($g = 1.6$ m s^{-2})?
8. HL A 0.50 kg mass on a spring ($k = 50$ N m^{-1}) is pulled 4.0 cm from equilibrium and released. Find ω, v_{max} and the total energy.

Show answers

1. acceleration ∝ displacement, and directed towards equilibrium
2. at the equilibrium position
3. it doubles
4. it doubles
5. $-\omega^2$
6. $\pi/2$
7. $T \propto 1/\sqrt{g}$
   $T = 2.0 \times \sqrt{(9.8/1.6)}$
   $= \mathbf{4.9\ s}$
8. $\omega = \sqrt{(k/m)} = 10\ \text{rad s}^{-1}$
   $v_{max} = \omega x_0 = \mathbf{0.40\ m\ s^{-1}}$
   $E_T = \frac{1}{2}kx_0^2 = \frac{1}{2} \times 50 \times 0.040^2 = \mathbf{0.040\ J}$

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