IB DP PHYSICS · THEME D FIELDS · D.1 · ~7 MIN READ

D.1 Gravitational fields

Kepler's three laws, Newton's law of gravitation, gravitational field strength and field lines, orbits, and (HL) gravitational potential and potential energy, equipotentials, escape speed, orbital speed and energy.

Kepler's laws | Newton's law of gravitation | Field strength g | Field lines | Orbits | Potential (HL) | Escape & orbital speed (HL)

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No tag = **SL and HL** HL ONLY HL students only DATA BOOKLET given in the exam

M1 A1 how marks are awarded highlight = comes up again and again Test yourself at the end

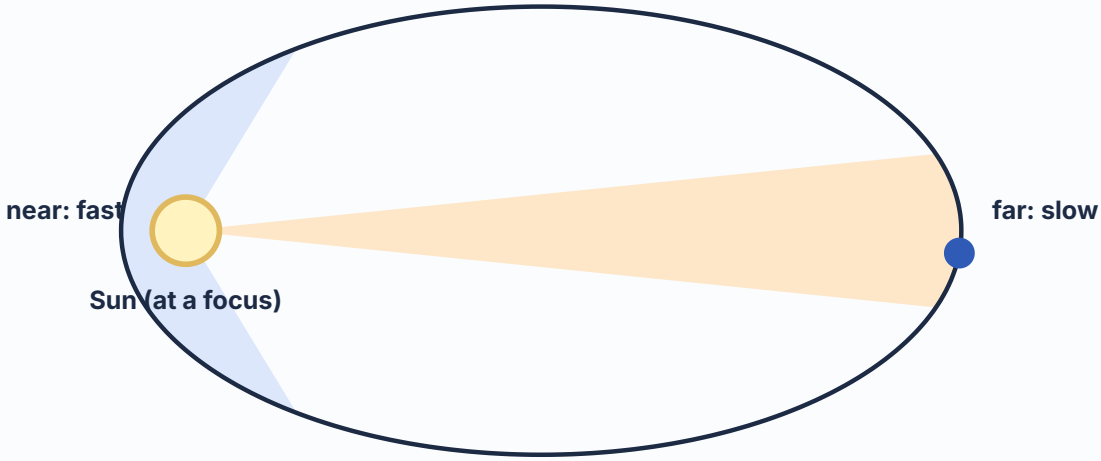
IN 30 SECONDS

- Every pair of masses attracts: $F = Gm_1m_2/r^2$. Spheres act as if all their mass were at the centre (for points outside them).
- **Field strength** $g = F/m = GM/r^2$, in N kg^{-1}. Field lines show the direction of the force on a test mass.
- **Kepler:** orbits are ellipses with the Sun at a focus; equal areas in equal times; $T^2 \propto r^3$, which follows from gravity providing the centripetal force.
- **HL:** gravitational potential $V_g = -GM/r$ (zero at infinity), potential energy $E_p = -GMm/r$, $g = -\Delta V/\Delta r$, escape speed $\sqrt{(2GM/r)}$ and orbital speed $\sqrt{(GM/r)}$.

BY THE END YOU CAN

- State Kepler's three laws and Newton's law of gravitation.
- Calculate gravitational forces and field strengths, and draw field lines.
- Derive and use $T^2 \propto r^3$ for circular orbits.
- Use gravitational potential, potential energy and work done in a field. HL
- Relate field strength to the potential gradient and sketch equipotentials. HL
- Calculate escape speed, orbital speed and orbital energy, and describe the effect of atmospheric drag. HL

1 Kepler and Newton



Equal areas swept in equal times (Kepler 2). $T^2 \propto r^3$ (Kepler 3).

The planet sweeps out equal areas in equal times, so it moves fastest when it is closest to the Sun.

- **Kepler 1:** planets move in ellipses with the Sun at one focus. **Kepler 2:** a line from the Sun to the planet sweeps equal areas in equal times. **Kepler 3:** $T^2 \propto r^3$ (r = mean orbital radius).

$$F = G\frac{m_1 m_2}{r^2}$$ DATA BOOKLET

G = 6.67×10^{-11} N m^2 kg^{-2}; r between centres

WORKED EXAMPLE

Find the gravitational force between the Earth (5.97×10^{24} kg) and the Moon (7.35×10^{22} kg), 3.84×10^8 m apart.

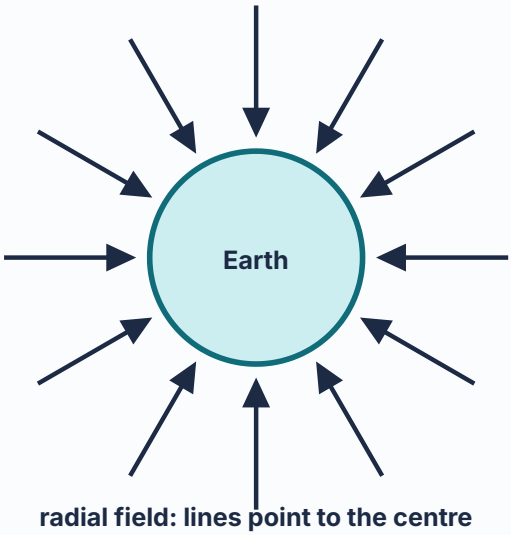
1. $F = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 7.35 \times 10^{22}/(3.84 \times 10^8)^2$ M1
2. **$= 2.0 \times 10^{20}$ N** A1

- **Point masses:** a uniform sphere behaves as a point mass at its centre for anything outside it. Two bodies whose separation is large compared with their sizes can also be treated as points.

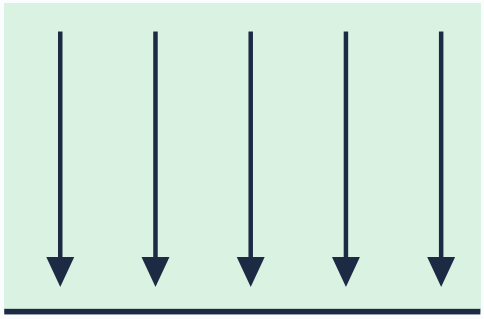
2 Gravitational field strength

$$g = \frac{F}{m} = \frac{GM}{r^2}$$ DATA BOOKLET

field strength, N kg^{-1} (the same as m s^{-2})

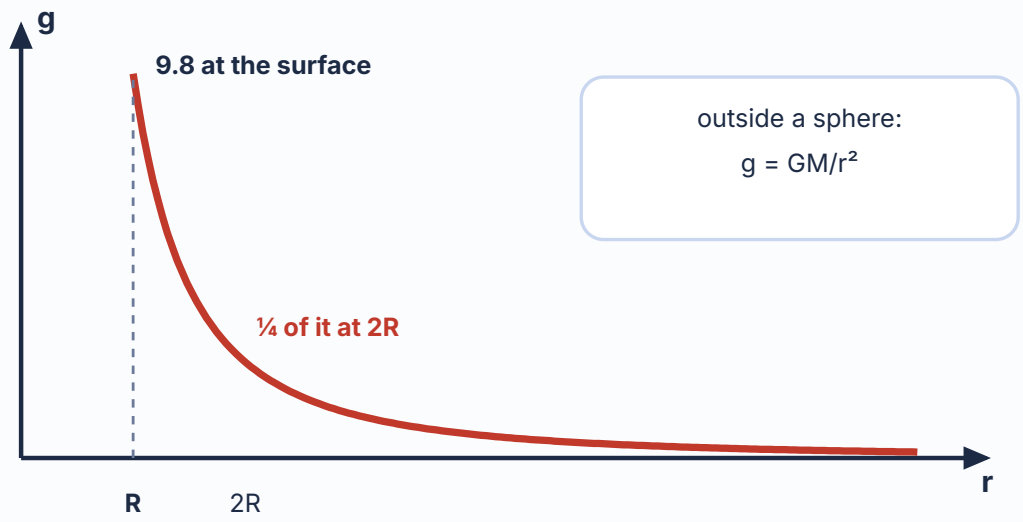


radial field: lines point to the centre



near the surface: almost uniform
(parallel, equally spaced)

Radial far away, nearly uniform close to the surface.



Outside the planet g falls as $1/r^2$. Double the distance from the centre and g drops to a quarter.

WORKED EXAMPLE

Find g at the Earth's surface ($R = 6.37 \times 10^6$ m) and at the height of the International Space Station (400 km).

1. surface: $g = GM/R^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24}/(6.37 \times 10^6)^2 =$ **$9.8\ \text{N kg}^{-1}$** A1
2. ISS: $r = 6.77 \times 10^6$ m, $g =$ **$8.7\ \text{N kg}^{-1}$** M1 A1
3. **astronauts float because they are in free fall with the station, not because g = 0**

COMMON MISTAKE

Using the height above the surface as r. In GM/r^2, r is measured from the **centre** of the planet.

3 Circular orbits

- **Gravity provides the centripetal force:** $GMm/r^2 = mv^2/r = m(4\pi^2 r/T^2)$. So **$T^2 = (4\pi^2/GM)\,r^3$**, Kepler's third law, and the orbiting mass m cancels.

WORKED EXAMPLE

Find the orbital radius of a geostationary satellite (T = 24 h).

1. $r^3 = GMT^2/4\pi^2 = 6.67 \times 10^{-11} \times 5.97 \times 10^{24} \times 86\,400^2/4\pi^2$ M1
2. **$r = 4.2 \times 10^7$ m (about 36 000 km above the surface)** A1

4 Gravitational potential and energy HL

Gravitational potential energy E_p — the work done in bringing the masses together from infinite separation. It is negative because gravity is attractive.

Gravitational potential V_g — the work done per unit mass in bringing a small test mass from infinity to that point. A scalar, zero at infinity.

$$E_p = -G\frac{m_1 m_2}{r}$$ DATA BOOKLET

J

$$V_g = -\frac{GM}{r}$$ DATA BOOKLET

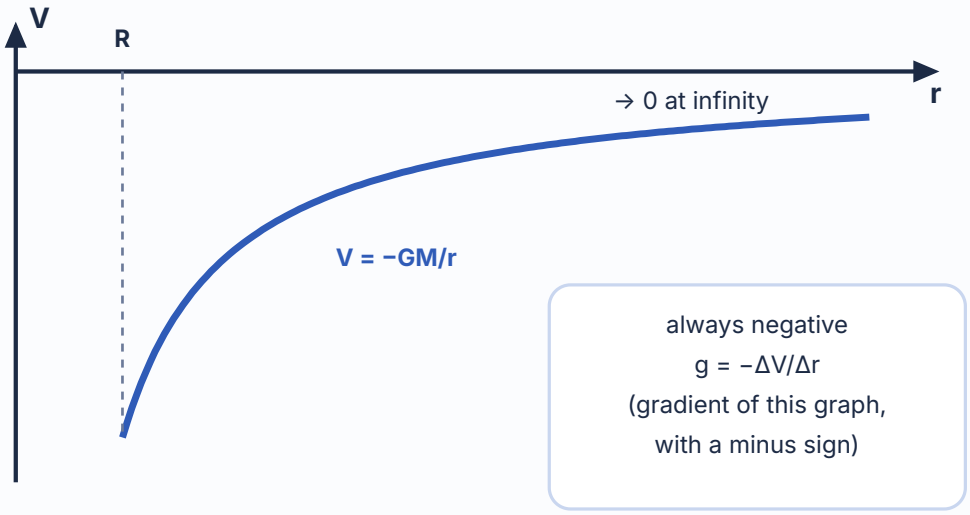
$J\ kg^{-1}$

$$g = -\frac{\Delta V_g}{\Delta r}$$ DATA BOOKLET

field strength is minus the potential gradient

$$W = m\Delta V_g$$ DATA BOOKLET

work done moving mass m between two points

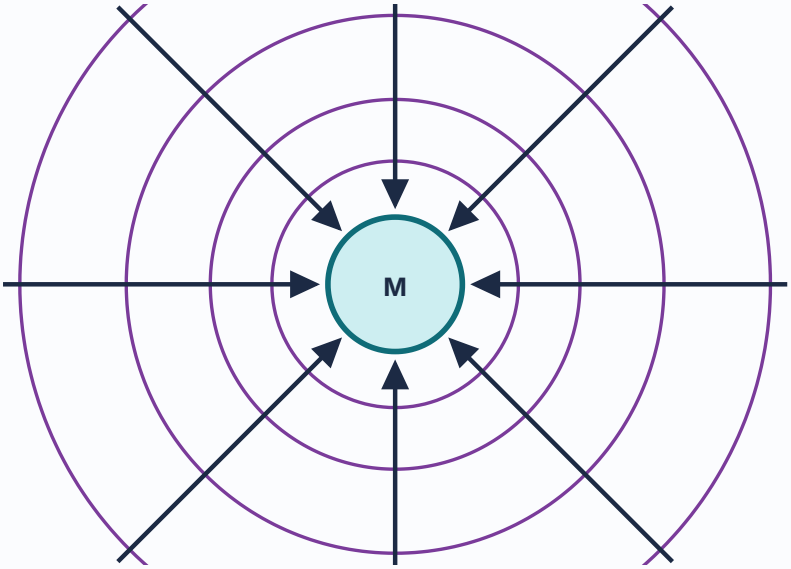


The potential rises towards zero as you move away; the slope of this curve gives g.

WORKED EXAMPLE

How much energy is needed to lift a 1000 kg satellite from the Earth's surface to 400 km (ignore the kinetic energy it needs)?

1. $\Delta V_g = GM(1/R - 1/r) = 3.98 \times 10^{14} \times (1/6.37 \times 10^6 - 1/6.77 \times 10^6)$ M1
2. $= 3.69 \times 10^6$ J kg^{-1} A1
3. **$W = m\Delta V = 3.7 \times 10^9$ J (mgh with g = 9.8 would give 3.9×10^9 J, an overestimate)** A1



equipotentials (purple) are spheres, always perpendicular to field lines
no work moving along one

Equipotentials are spaced further apart further out, because the field is weaker there.

- **Equipotential surfaces** join points of equal potential. They are always perpendicular to field lines, and no work is done moving along one.

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5 Escape speed and orbital speed HL

$$v_{esc} = \sqrt{\frac{2GM}{r}}$$ DATA BOOKLET

minimum launch speed to reach infinity (no air resistance)

$$v_{orbital} = \sqrt{\frac{GM}{r}}$$ DATA BOOKLET

speed of a circular orbit of radius r

WORKED EXAMPLE

Find the escape speed from the Earth's surface and the orbital speed of the ISS.

1. $v_{esc} = \sqrt{(2 \times 3.98 \times 10^{14}/6.37 \times 10^{6})}$ = **11.2 km s^{-1}** A1
2. **$v_{orb} = \sqrt{(3.98 \times 10^{14}/6.77 \times 10^{6})} = 7.7$ km s^{-1}** A1

- **Orbital energy:** $E_k = GMm/2r$, $E_p = -GMm/r$, so the total $E = -GMm/2r$. A lower orbit has more negative total energy but **higher** speed.
- **Atmospheric drag** on a low satellite removes energy. Its total energy becomes more negative, so it falls to a lower orbit, where it actually speeds up, until it re-enters the atmosphere.

COMMON MISTAKE

Deriving escape speed from the orbital speed. They are different: $v_{esc} = \sqrt{2} \times v_{orbital}$ at the same radius.

EXAM TIP

For "show that" orbital questions, start from $GMm/r^2 = mv^2/r$ and cancel m. It is worth one method mark on its own.

LINKING QUESTION

The inverse-square law here has exactly the same mathematics as Coulomb's law in D.2: compare $g = GM/r^2$ with $E = kQ/r^2$, and $V_g = -GM/r$ with $V_e = kQ/r$.

IDEA FOR YOUR IA / EXPLORATION

Use orbital data for Jupiter's moons (from a planetarium app or published tables) to test Kepler's third law with a log–log graph, and find Jupiter's mass from the gradient.

Test yourself — cover the answers first

QUICK CHECK

1. State Newton's law of gravitation.
2. What is the unit of g?
3. If r doubles, what happens to g?
4. Why is a geostationary satellite always above the same point?
5. HL Why is gravitational potential negative?
6. HL What is the ratio $v_{esc}/v_{orbital}$ at the same radius?

EXAM-STYLE

7. A planet has twice the Earth's mass and twice its radius. Find g at its surface.
8. HL A 500 kg satellite orbits at $r = 8.0 \times 10^6$ m around the Earth ($GM = 3.98 \times 10^{14}$ m^3 s^{-2}). Find its speed, period and total energy.

Show answers

1. $F = Gm_1m_2/r^2$, attractive, along the line between centres
2. N kg^{-1} (= m s^{-2})
3. it falls to a quarter
4. its period equals the Earth's rotation period (24 h) and it orbits above the equator
5. it is zero at infinity and work must be done to move a mass out to infinity
6. √2
7. $g \propto M/R^2$
   $= 9.8 \times 2/4$
   $=$ **4.9 N kg^{-1}**
8. $v = \sqrt{(GM/r)} =$ **7.05 km s^{-1}**
   $T = 2\pi r/v =$ **7.1×10^3 s** (about 2 h)
   $E = -GMm/2r = -3.98 \times 10^{14} \times 500/(1.6 \times 10^7) =$ **-1.2×10^{10} J**

Stuck on a question?

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