IGCSE MATHEMATICS 0580 · CHAPTER 7 NOTES · ~31 MIN READ

Geometry & circle theorems

Every angle rule in one place: lines, parallel lines, triangles, polygons, symmetry, congruence, similarity, constructions and all the circle theorems. Each rule has a picture, a worked example and the exact reason to write in the exam.

Angle facts | Polygons | Similarity & congruence | Symmetry | Circle theorems

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No tag = **Core** (everyone) EXTENDED Extended only highlight = comes up again and again

Test yourself at the end

IN 30 SECONDS

- Angles obey a few **fixed totals**: **180°** on a line and in a triangle, **360°** around a point and in a quadrilateral. Parallel lines give **equal** (corresponding, alternate) and **supplementary** (co-interior) angles.
- **Polygons:** interior angles add to **(n – 2) × 180°** and exterior angles always add to **360°**. **Similar** shapes have the same angles and a scale factor **k**: areas scale by k^2 and volumes by k^3.
- **Circle theorems** (Extended): angle at the centre = **2×** angle at the edge, angles in a semicircle = **90°**, opposite angles of a cyclic quadrilateral = **180°**, tangent ⊥ radius. **Always give the reason.**

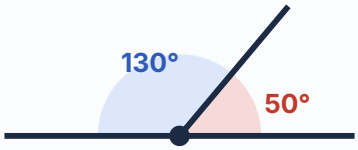
BY THE END YOU CAN

- Find missing angles using lines, parallel lines, triangles and quadrilaterals, and write the reason.
- Calculate interior and exterior angles of polygons.
- Find lines of symmetry and the order of rotational symmetry.
- Recognise congruent shapes and use similarity to find missing lengths.
- Construct triangles, bisectors and loci with a ruler and compasses.
- Use the area and volume scale factors for similar shapes. EXT
- Use all the circle theorems to find angles and give reasons. EXT

1 Angle facts

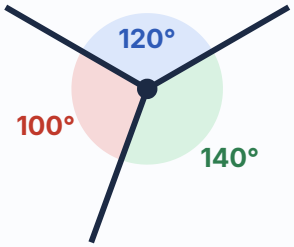
Lines and points

On a straight line



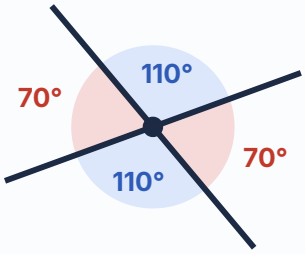
angles on a line add up to 180°

Around a point



angles around a point add up to 360°

Vertically opposite



opposite angles are equal

Three facts cover most angle questions: a line is 180°, a full turn is 360°, and vertically opposite angles are equal.

Angle types — **acute** (less than 90°), **right** (exactly 90°), **obtuse** (between 90° and 180°), **straight** (180°) and **reflex** (between 180° and 360°).

- **Angles on a straight line** add up to **180°**. **Angles around a point** add up to **360°**.
- **Vertically opposite angles** (the opposite angles where two lines cross) are **equal**.
- **Always write a reason** next to each step in a "find the angle" question. The reason is worth a mark.

WORKED EXAMPLE

Angles on a straight line are 3x, (2x + 10) and 70°. Find x.

1. angles on a straight line add up to 180°
2. 3x + 2x + 10 + 70 = 180
3. 5x + 80 = 180, so 5x = 100
4. **x = 20**
5. **the angles are 60°, 50° and 70° (sum 180° ✓)**

WORKED EXAMPLE

Three angles around a point are 90°, 130° and 2y. Find y.

1. angles around a point add up to 360°
2. 90 + 130 + 2y = 360
3. 2y = 140
4. **y = 70**

WORKED EXAMPLE

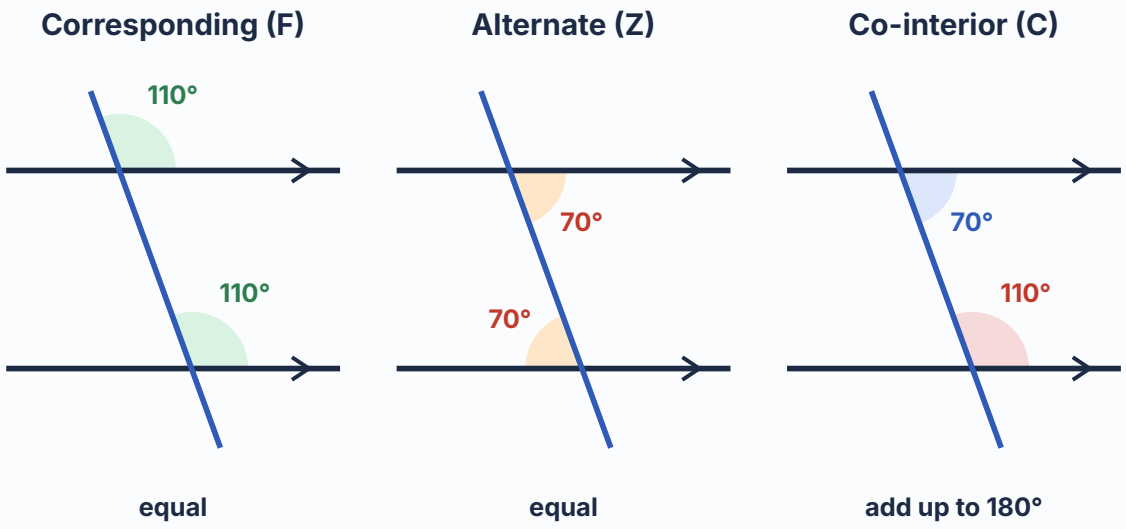
Two straight lines cross. Opposite angles are (4x – 10)° and (2x + 30)°. Find x and the angle.

1. vertically opposite angles are equal
2. 4x – 10 = 2x + 30
3. 2x = 40, so **x = 20**
4. **angle = 4(20) – 10 = 70°**

COMMON MISTAKE

Writing "they add up to 180" with no reason. The marks are for the **named rule**: "angles on a straight line add up to 180°".

Parallel lines



A line crossing two parallel lines makes F, Z and C shapes. F and Z angles are equal. C angles add up to 180°.

Parallel lines — lines that stay the same distance apart and never meet. Arrows on the lines show they are parallel.

- **Corresponding angles** (F shape) are **equal**.
- **Alternate angles** (Z shape) are **equal**.
- **Co-interior angles** (C or U shape) add up to **180°**.
- These rules work **only if the lines are parallel**. If the question does not say so (or show arrows), you cannot use them.

WORKED EXAMPLE

AB is parallel to CD. One angle where a transversal crosses AB is 125°. Find the alternate, corresponding and co-interior angles at CD.

1. alternate angle = **125°** (alternate angles are equal)
2. corresponding angle = **125°** (corresponding angles are equal)
3. **co-interior angle = 180° − 125° = 55° (co-interior angles add up to 180°)**

WORKED EXAMPLE

Two co-interior angles are x and (2x − 15). Find both angles.

1. co-interior angles add up to 180°
2. x + 2x − 15 = 180
3. 3x = 195, so **x = 65**
4. **the angles are 65° and 115° (65 + 115 = 180 ✓)**

COMMON MISTAKE

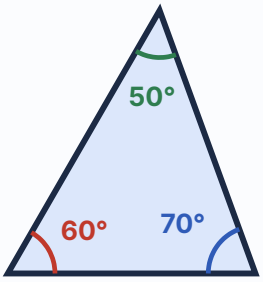
Calling an angle "alternate" when it is really **co-interior**. Trace the shape: a **Z** joins equal angles inside the parallel lines, a **C** joins angles that add up to **180°**.

EXAM TIP

Extend the lines on the diagram with your pencil until you can see the F, Z or C. Mark the angles you know in a colour and fill in the rest one by one.

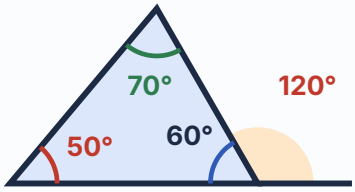
Triangles

Angle sum



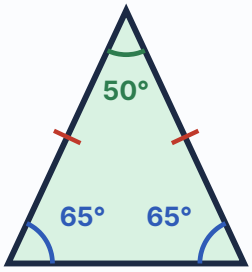
angles add up to 180°
60 + 70 + 50 = 180

Exterior angle



exterior = sum of the two opposite interior
120° = 50° + 70°

Isosceles



equal sides give equal base angles
65 + 65 + 50 = 180

The three angles of any triangle add up to 180°. An exterior angle equals the sum of the two interior angles opposite to it.

- **Angles in a triangle** add up to **180°**.
- **Exterior angle of a triangle** = the sum of the **two opposite interior angles** (here 120° = 50° + 70°).
- **Isosceles:** two equal sides and two equal base angles. **Equilateral:** three equal sides and three angles of 60°.

Triangle	Sides	Angles
Equilateral	all 3 equal	all 60°
Isosceles	2 equal	2 equal base angles
Scalene	none equal	none equal
Right-angled	any	one angle is 90°

WORKED EXAMPLE

The angles of a triangle are x, 2x and 3x. Find them.

1. x + 2x + 3x = 180
2. 6x = 180, so x = 30
3. **the angles are 30°, 60° and 90° (a right-angled triangle)**

WORKED EXAMPLE

An isosceles triangle has an apex angle of 40°. Find the base angles.

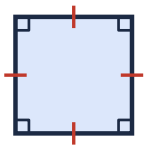
1. the two base angles are equal
2. 180 − 40 = 140
3. **140 ÷ 2 = 70° each**

WORKED EXAMPLE

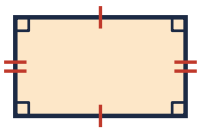
The exterior angle at C of triangle ABC is 125° and angle A is 60°. Find angle B.

1. exterior angle = sum of the two opposite interior angles
2. 125 = 60 + B
3. **B = 65°**
4. **check: interior C = 180 − 125 = 55° and 60 + 65 + 55 = 180 ✓**

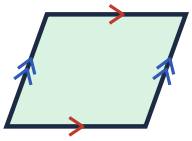
Quadrilaterals



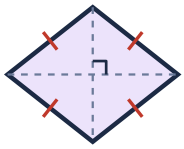
Square
4 equal sides
4 right angles



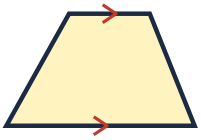
Rectangle
opposite sides equal
4 right angles



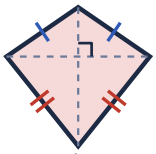
Parallelogram
opposite sides parallel
opposite angles equal



Rhombus
4 equal sides
diagonals cross at 90°



Trapezium
one pair of
parallel sides



Kite
2 pairs of equal sides
diagonals cross at 90°

The four angles of any quadrilateral add up to 360°. Learn the properties of each special quadrilateral.

- **Angles in a quadrilateral** add up to **360°**. (Split it into two triangles: 2 × 180° = 360°.)
- **Parallelogram:** opposite angles are equal and **co-interior angles add up to 180°**. **Kite:** one pair of opposite angles is equal.
- **Diagonals:** a rectangle's are **equal** and bisect each other. A rhombus's and a kite's **cross at 90°**.

WORKED EXAMPLE

Four angles of a quadrilateral are 85°, 110°, x and 2x. Find x.

1. angles in a quadrilateral add up to 360°
2. 85 + 110 + x + 2x = 360
3. 195 + 3x = 360, so 3x = 165
4. **x = 55, and the angles are 85°, 110°, 55° and 110°**

WORKED EXAMPLE

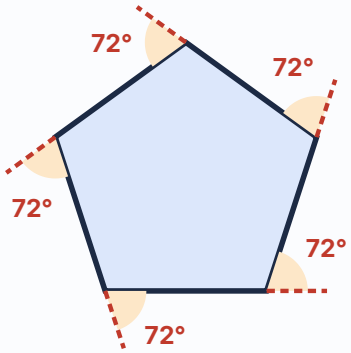
One angle of a parallelogram is 70°. Find the other three angles.

1. opposite angle = **70°** (opposite angles are equal)
2. **the two other angles are co-interior with 70°: 180 − 70 = 110° each**

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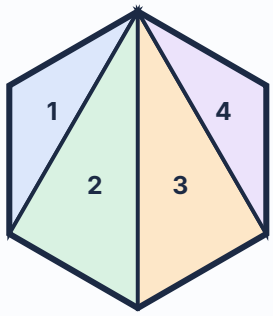
Polygons

Exterior angles add up to 360°



exterior angle = 360° ÷ 5 = 72°
interior angle = 180° − 72° = 108°

Interior: split into triangles



6 sides give 4 triangles
sum = (6 − 2) × 180° = 720°

Walking round a polygon you turn through all the exterior angles and finish facing the same way: one full turn of 360°.

Polygon — a closed shape with straight sides. A **regular** polygon has all sides equal and all angles equal.

sum of interior angles = $(n - 2) \times 180°$

n = number of sides

regular polygon: exterior angle = $360° \div n$

then interior = 180° − exterior

Sides	Name	Sides	Name
5	pentagon	8	octagon
6	hexagon	9	nonagon
7	heptagon	10	decagon

- **Exterior angles of any polygon add up to 360°.** The interior angle and the exterior angle at one vertex add up to **180°**.
- **Going backwards:** n = 360 ÷ (exterior angle). An exterior angle must **divide 360 exactly**.

WORKED EXAMPLE

Find the sum of the interior angles of an octagon and the size of each angle of a regular octagon.

1. sum = (8 − 2) × 180 = 6 × 180 = **1080°**
2. regular: exterior angle = 360 ÷ 8 = 45°
3. **interior angle = 180 − 45 = 135° (or 1080 ÷ 8 = 135°)**

WORKED EXAMPLE

A regular polygon has an exterior angle of 20°. How many sides does it have?

1. **n = 360 ÷ 20 = 18 sides**

WORKED EXAMPLE

Each interior angle of a regular polygon is 156°. Find the number of sides.

1. exterior angle = 180 − 156 = 24°
2. **n = 360 ÷ 24 = 15 sides**

WORKED EXAMPLE

Four angles of a pentagon are 100°, 110°, 120° and 95°. Find the fifth angle.

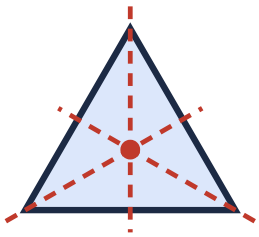
1. sum of interior angles = (5 − 2) × 180 = 540°
2. 100 + 110 + 120 + 95 = 425
3. **fifth angle = 540 − 425 = 115°**

COMMON MISTAKE

Dividing 180° by n instead of 360° by n. 180 ÷ n is **not** an angle of the polygon. Use **360 ÷ n for the exterior angle**, then subtract from 180.

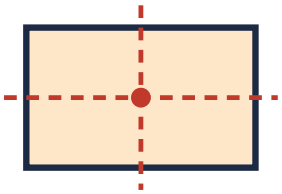
2 Symmetry

Line and rotational symmetry



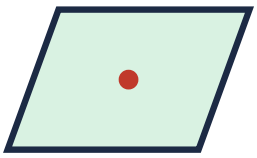
Equilateral triangle

lines of symmetry: 3
rotational order: 3



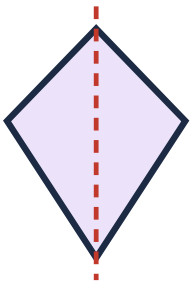
Rectangle

lines of symmetry: 2
rotational order: 2



Parallelogram

lines of symmetry: 0
rotational order: 2



Kite

lines of symmetry: 1
rotational order: 1

A shape has line symmetry if a mirror line splits it into two matching halves, and rotational symmetry if it fits on itself during a turn.

Line of symmetry — a mirror line. One half is the **reflection** of the other.

Order of rotational symmetry — the number of times a shape fits onto itself in one full turn of 360°. Order 1 means no rotational symmetry.

Shape	Lines of symmetry	Order of rotation
Equilateral triangle	3	3
Square	4	4
Rectangle	2	2
Rhombus	2	2
Parallelogram	0	2
Kite	1	1
Isosceles trapezium	1	1
Regular n-sided polygon	n	n

- A **regular polygon** with n sides has **n lines of symmetry** and rotational symmetry of **order n**.
- **Symmetry in letters and patterns:** the letter H has 2 lines and order 2. The letter S has 0 lines but order 2.

WORKED EXAMPLE

Describe the symmetry of a regular pentagon.

1. it has **5 lines of symmetry** (each from a vertex to the middle of the opposite side)
2. **rotational symmetry of order 5 (turns of 72°)**

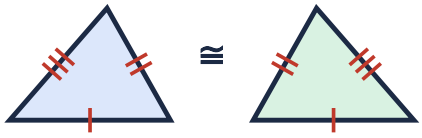
COMMON MISTAKE

Giving a **parallelogram** lines of symmetry. A (non-rectangle, non-rhombus) parallelogram has **no** lines of symmetry, but it has rotational symmetry of **order 2**.

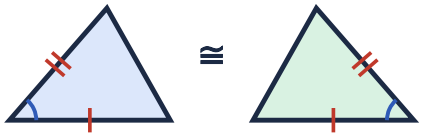
3 Congruence and similarity

Congruent shapes

SSS 3 sides equal



SAS 2 sides and the angle between



ASA 2 angles and a side



RHS right angle, hypotenuse, 1 side



Two triangles are congruent if you can show one of four sets of matching facts. They are identical in shape and size.

Congruent — exactly the same shape and size. One can be moved onto the other by a reflection, rotation or translation.

- **SSS:** three sides equal. **SAS:** two sides and the **angle between** them. **ASA:** two angles and a side. **RHS:** a right angle, the hypotenuse and one other side.
- **AAA is not enough.** Equal angles only prove the triangles are **similar**, not the same size. **SSA** (two sides and a non-included angle) is also not enough.

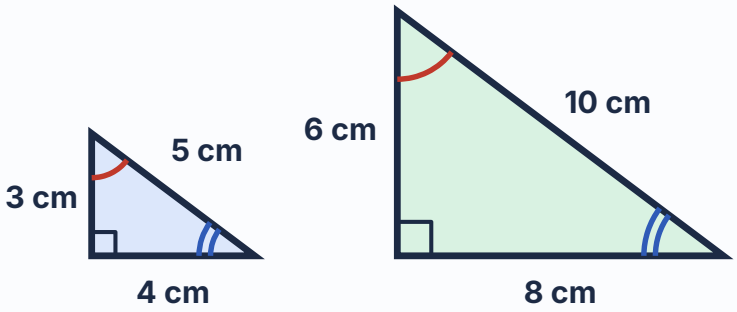
WORKED EXAMPLE

EXT **ABCD is a parallelogram. Show that triangles ABC and CDA are congruent.**

1. AB = CD (opposite sides of a parallelogram are equal)
2. BC = DA (opposite sides are equal)
3. AC is a **common side**
4. **three pairs of equal sides: the triangles are congruent by SSS**

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Similar shapes



Similar triangles

same angles,
sides in the same ratio
scale factor k
= 6 ÷ 3 = 8 ÷ 4 = 2
small to big: × 2
big to small: ÷ 2

Similar shapes have the same angles and every length is multiplied by the same scale factor.

Similar — same shape but a different size. All **angles are equal** and all **lengths are in the same ratio**.

$$\text{scale factor k} = \frac{\text{length on the new shape}}{\text{corresponding length on the old shape}}$$

always match **corresponding** sides

- **Find k first,** using a pair of sides you know. Then multiply (small to big) or divide (big to small).
- **Parallel lines make similar triangles:** corresponding angles are equal, so a line parallel to one side of a triangle makes a smaller similar triangle.

WORKED EXAMPLE

Triangle ABC is similar to triangle PQR. AB = 6 cm, PQ = 9 cm and BC = 8 cm. Find QR.

1. scale factor k = 9 ÷ 6 = 1.5
2. QR corresponds to BC
3. **QR = 8 × 1.5 = 12 cm**

WORKED EXAMPLE

A 1.8 m person casts a shadow of 2.4 m. At the same time a tree casts a shadow of 10 m. Find the height of the tree.

1. the triangles are similar: $\frac{\text{tree}}{1.8} = \frac{10}{2.4}$
2. tree = 1.8 × 10 ÷ 2.4
3. **7.5 m**

WORKED EXAMPLE

In triangle ABC, DE is parallel to BC with D on AB and E on AC. AD = 4 cm, DB = 2 cm and DE = 5 cm. Find BC.

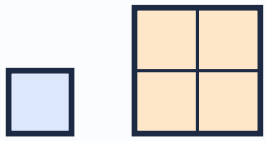
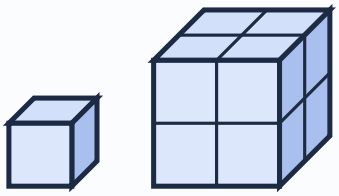
1. triangles ADE and ABC are similar (corresponding angles)
2. AB = 4 + 2 = 6 cm
3. k = 6 ÷ 4 = 1.5
4. **BC = 5 × 1.5 = 7.5 cm**

COMMON MISTAKE

Using **AD and DB** as the pair of corresponding sides. The big triangle's side is **AB = AD + DB**, not DB. Always use the **whole side**.

Area and volume scale factors

EXTENDED

Length: × k	Area: × k^2	Volume: × k^3
		
1 2	area 1 area 4	volume 1 volume 8
k = 2 lengths × 2	k = 2 area × 2^2 = × 4	k = 2 volume × 2^3 = × 8

area scale factor = k^2 | volume scale factor = k^3

Double the lengths and the area becomes 4 times as big and the volume 8 times as big.

Scale factors — if the **length scale factor is k**, the **area scale factor is k^2** and the **volume scale factor is k^3**.

- EXT **Go in the right direction:** to find k from areas take a **square root**. To find k from volumes take a **cube root**.
- EXT Areas of similar shapes, surface areas and the areas of faces all use **k^2**. Volumes, capacities and masses (same material) use **k^3**.

WORKED EXAMPLE

EXT **Two similar cones have heights 4 cm and 6 cm. The small cone has a surface area of 80 cm^2 and a volume of 64 cm^3. Find the surface area and volume of the large cone.**

1. k = 6 ÷ 4 = 1.5
2. area scale factor = 1.5^2 = 2.25, so surface area = 80 × 2.25 = **180 cm^2**
3. **volume scale factor = 1.5^3 = 3.375, so volume = 64 × 3.375 = 216 cm^3**

WORKED EXAMPLE

EXT **Two similar bottles have capacities 200 ml and 1600 ml. The small one is 10 cm tall. Find the height of the large bottle.**

1. volume scale factor = 1600 ÷ 200 = 8
2. k = $\sqrt[3]{8}$ = 2
3. **height = 10 × 2 = 20 cm**

WORKED EXAMPLE

EXT **The areas of two similar shapes are 12 cm^2 and 27 cm^2. Find the ratio of their lengths and of their volumes (for similar solids).**

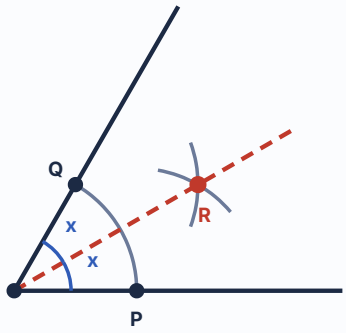
1. area scale factor = 27 ÷ 12 = 2.25
2. k = √2.25 = 1.5, so the lengths are in the ratio **2 : 3**
3. **volume scale factor = 1.5^3 = 3.375, so volumes are in the ratio 8 : 27**

COMMON MISTAKE

EXT Multiplying an area by the length scale factor. If the lengths double, the area is **×4**, not ×2.

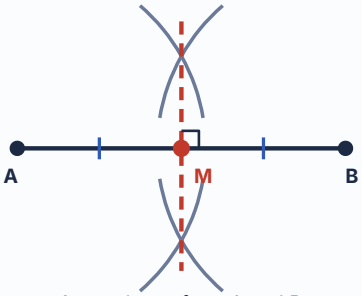
Constructions with a ruler and compasses

Bisect an angle



1. arc from O cuts both arms
2. equal arcs from P and Q cross at R
3. draw the line OR

Perpendicular bisector



1. equal arcs from A and B
2. the arcs cross above and below
3. join the crossing points

Construction arcs show your method, so never rub them out. Keep the compasses at the same width for each pair of arcs.

- **Draw a triangle from three sides (SSS):** (1) draw the longest side as the base; (2) arc from one end with radius = second side; (3) arc from the other end with radius = third side; (4) join the base ends to where the arcs cross.
- **Angle bisector:** arcs from O cut both arms, equal arcs from those two points cross, join O to the crossing point.
- **Perpendicular bisector:** equal arcs (radius more than **half** AB) from A and from B cross above and below, and the line through the crossings is the bisector.

WORKED EXAMPLE

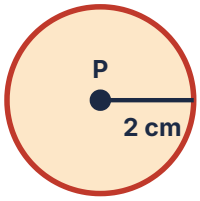
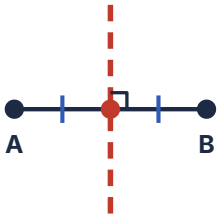
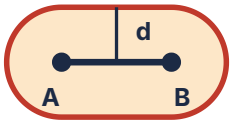
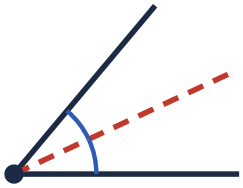
Construct triangle ABC with AB = 7 cm, BC = 5 cm and AC = 6 cm.

1. draw AB = 7 cm with a ruler
2. set the compasses to 6 cm, centre A, draw an arc above AB
3. set the compasses to 5 cm, centre B, draw an arc to cross the first arc: this is **C**
4. **join AC and BC**

EXAM TIP

Use a **sharp pencil** and place the compass point **carefully**: a slip of 1 mm moves the crossing point. Measure to check: your angles should add up to 180°.

Loci

Fixed distance from a point	Equal distance from 2 points	Fixed distance from a line	Equal distance from 2 lines
			
a circle	**the perpendicular bisector**	**two parallel lines and two semicircles**	**the angle bisector**

A locus is a path or a region. Learn the four standard loci and combine them.

Locus — the set of **all points** that fit a rule. The plural is **loci**.

- **Fixed distance from a point:** a **circle**. **Equal distance from two points:** the **perpendicular bisector**.
- **Fixed distance from a line:** two **parallel lines** joined by **semicircles** at the ends. **Equal distance from two lines:** the **angle bisector**.
- **Regions:** "**closer to A than to B**" is one side of the perpendicular bisector. "**within 3 cm of C**" is inside the circle. Shade the part that satisfies **all** the rules.

WORKED EXAMPLE

A and B are 6 cm apart. Show the region that is closer to A than to B and within 4 cm of A.

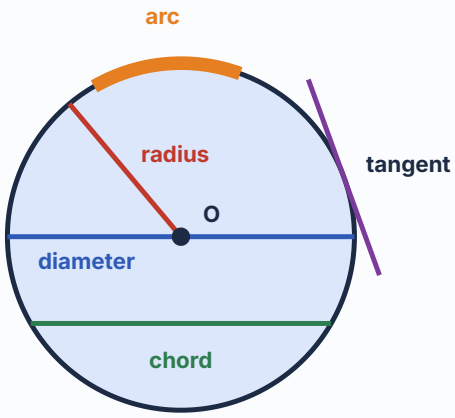
1. draw the perpendicular bisector of AB (it passes 3 cm from A)
2. draw a circle of radius 4 cm with centre A
3. the region is on **A's side** of the bisector and **inside** the circle
4. **shade it**

Riyaz Sir

5 Circles and circle theorems

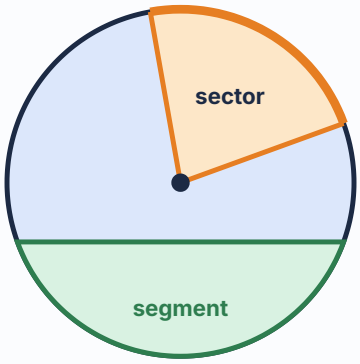
Parts of a circle

Parts of a circle



circumference: the whole curved edge

Sector and segment



sector: two radii and an arc
segment: a chord and an arc

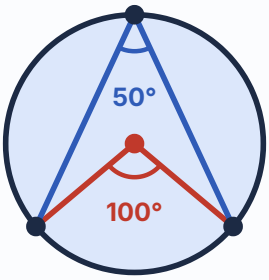
Learn the eight words: radius, diameter, circumference, chord, tangent, arc, sector, segment.

- **Diameter = 2 × radius.** A diameter is the longest chord and it passes through the centre.
- **Arcs:** the shorter one is the **minor arc** and the longer one is the **major arc**. A **tangent** touches the circle at exactly **one point**.
- **Sector** = a slice between two radii. **Segment** = the region between a chord and the arc.

Angle at the centre, same segment, semicircle

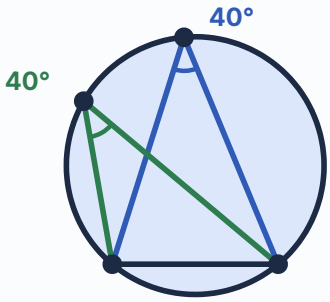
EXTENDED

Angle at the centre



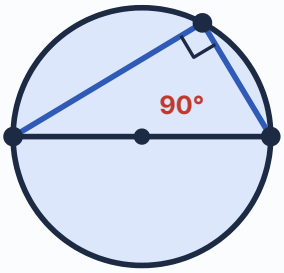
angle at the centre = 2 × angle at the edge

Same segment



angles in the same segment are equal

In a semicircle



angle in a semicircle = 90°

Three theorems about angles made by the same chord or arc.

Theorem 1 — the **angle at the centre** is **twice** the angle at the circumference (when both are subtended by the same arc).

- EXT **Angles in the same segment** are **equal**. Both angles stand on the **same chord** and are on the **same side** of it.
- EXT **Angle in a semicircle = 90°.** The chord is a **diameter**. (This is the special case of the first theorem: 2 × 90° = 180°.)

WORKED EXAMPLE

EXT **A, B and C are points on a circle with centre O. Angle ACB = 38°. Find angle AOB.**

1. the angle at the centre is twice the angle at the circumference
2. **AOB = 2 × 38 = 76°**

WORKED EXAMPLE

EXT **AB is a diameter of a circle and C is on the circle. Angle BAC = 35°. Find angle ACB and angle ABC.**

1. angle in a semicircle: **ACB = 90°**
2. **angles in a triangle: ABC = 180 − 90 − 35 = 55°**

WORKED EXAMPLE

EXT **The chord AB makes angle ACB = 48° with C on the circle. D is another point on the same side of AB. Find angle ADB.**

1. angles in the same segment are equal
2. **ADB = 48°**

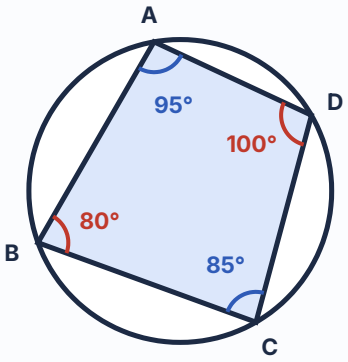
COMMON MISTAKE

EXT Using the centre rule when the angle is **not at the centre**. Check that the vertex of the bigger angle is at **O**, the **centre**.

Cyclic quadrilaterals

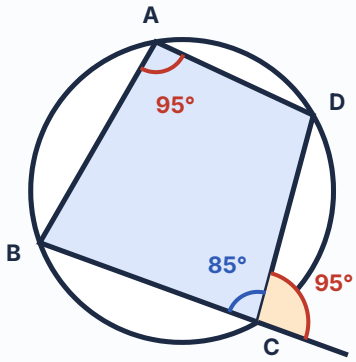
EXTENDED

Cyclic quadrilateral



opposite angles add up to 180°
95° + 85° = 180°
80° + 100° = 180°

Exterior angle



exterior angle = opposite interior angle
95° = 95°

If all four corners of a quadrilateral lie on a circle, opposite angles add up to 180°.

Cyclic quadrilateral — a quadrilateral with **all four vertices on a circle**.

- EXT **Opposite angles add up to 180°.**
- EXT An **exterior angle** equals the **interior opposite angle**. (Both are 180° minus the same angle.)

WORKED EXAMPLE

EXT **ABCD is a cyclic quadrilateral and angle ABC = 112°. Find angle ADC.**

1. opposite angles of a cyclic quadrilateral add up to 180°
2. **ADC = 180 − 112 = 68°**

WORKED EXAMPLE

EXT **Opposite angles of a cyclic quadrilateral are 3x and (2x + 20). Find them.**

1. they add up to 180°
2. 3x + 2x + 20 = 180
3. 5x = 160, so **x = 32**
4. **the angles are 96° and 84° (sum 180° ✓)**

WORKED EXAMPLE

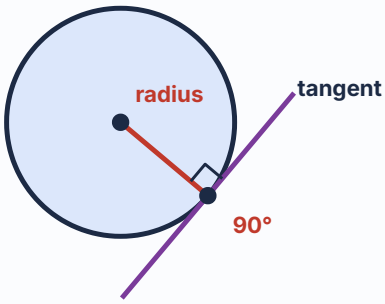
EXT **A cyclic quadrilateral has an exterior angle of 95° at C. Find the interior angle at A.**

1. the exterior angle equals the opposite interior angle
2. **angle at A = 95°**

Tangents

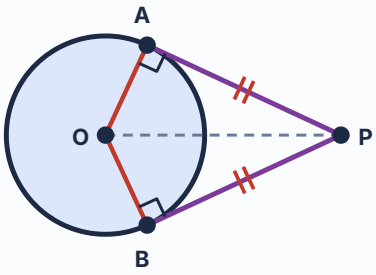
EXTENDED

Tangent and radius



a tangent meets the radius at exactly 90°

Two tangents from a point



two tangents from a point are equal: PA = PB

(triangle OAP and OBP are congruent: RHS)

A tangent meets the radius at 90°. The two tangents from an outside point are the same length.

Tangent — a straight line that **touches** a circle at **exactly one point**.

- EXT **A tangent is perpendicular to the radius** at the point of contact: the angle is **90°**.
- EXT **Two tangents drawn from one point are equal in length.** The shape OAPB is a **kite**.

WORKED EXAMPLE

EXT **PT is a tangent to a circle with centre O. Angle TOP = 52°. Find angle OPT.**

1. a tangent is perpendicular to the radius: angle OTP = 90°
2. **angles in a triangle: OPT = 180 − 90 − 52 = 38°**

WORKED EXAMPLE

EXT **Tangents PA and PB meet the circle at A and B. Angle APB = 50°. Find angle PAB.**

1. tangents from a point are equal: PA = PB, so triangle PAB is isosceles
2. **PAB = (180 − 50) ÷ 2 = 65°**

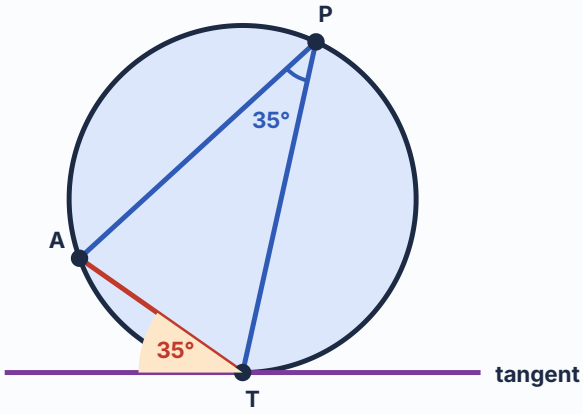
WORKED EXAMPLE

EXT **The radius of a circle is 5 cm and the centre O is 13 cm from an outside point P. Find the length of the tangent PT.**

1. OT ⊥ PT, so triangle OTP is right-angled at T
2. $PT^2 = 13^2 - 5^2 = 169 - 25 = 144$
3. **PT = 12 cm**

Alternate segment theorem

EXTENDED



Alternate segment theorem

the angle between a tangent
and a chord at T
= the angle in the
alternate segment
35° at T (tangent and chord)
35° at P (other segment)

The angle between a tangent and a chord equals the angle in the segment on the other side of the chord.

Alternate segment theorem — the angle between a **tangent and a chord** equals the angle in the **alternate segment** (the segment on the opposite side of the chord).

WORKED EXAMPLE

EXT **A tangent at T touches a circle. TA is a chord and the angle between the tangent and TA is 62°. P is on the circle in the alternate segment. Find angle TPA.**

1. alternate segment theorem
2. **TPA = 62°**

WORKED EXAMPLE

EXT **Triangle ABC is inside a circle. The tangent at A makes an angle of 48° with the chord AB. Find angle ACB.**

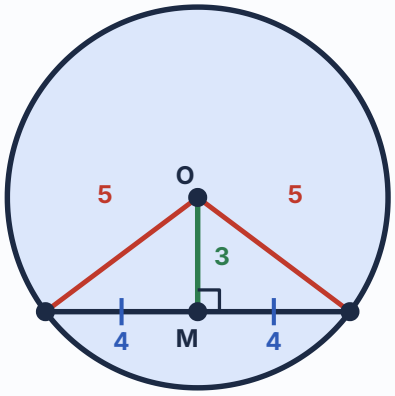
1. the angle between the tangent and the chord AB equals the angle in the alternate segment
2. **ACB = 48°**

MEMORY TRICK

Tangent-chord angle = the angle at the far edge. Find the chord, then look to the **other side** of the chord to find the equal angle.

Chords

EXTENDED



Chord and centre

The perpendicular from the centre to a chord bisects the chord.
Pythagoras in triangle OMA:
$OM^2 + AM^2 = OA^2$
$3^2 + 4^2 = 5^2$
chord AB = 4 + 4 = 8 cm

The perpendicular from the centre to a chord cuts the chord in half, which makes a right-angled triangle for Pythagoras.

- EXT The **perpendicular from the centre** to a chord **bisects** the chord. Equal chords are the same distance from the centre.
- EXT **Pythagoras:** radius² = (distance from the centre)² + (half the chord)².

WORKED EXAMPLE

EXT **A chord of length 24 cm is in a circle of radius 13 cm. Find the distance of the chord from the centre.**

1. the perpendicular bisects the chord: half chord = 12 cm
2. distance² = $13^2 - 12^2 = 169 - 144 = 25$
3. **distance = 5 cm**

Reasons to write

EXTENDED

- EXT Quote the **theorem in words** after each angle you find. These phrases get the method marks.

Reason	What it says
Angles on a straight line	add up to 180°
Angles around a point	add up to 360°
Vertically opposite angles	are equal
Corresponding or alternate angles	are equal (parallel lines)
Co-interior angles	add up to 180° (parallel lines)
Angles in a triangle	add up to 180°
Exterior angle of a triangle	equals the two opposite interior angles
Angle at the centre	is twice the angle at the circumference
Angles in the same segment	are equal
Angle in a semicircle	is 90°
Opposite angles in a cyclic quadrilateral	add up to 180°
Tangent and radius	meet at 90°
Alternate segment theorem	tangent-chord angle equals the angle in the other segment

Test yourself — cover the answers first

QUICK CHECK

1. Angles on a straight line are 2x, 3x and 50°. Find x.
2. Three angles around a point are 100°, 115° and a. Find a.
3. One co-interior angle is 72°. What is the other?
4. What do the angles of a quadrilateral add up to?
5. Two angles of a triangle are 48° and 67°. Find the third.
6. An isosceles triangle has an apex angle of 36°. Find a base angle.
7. Find the sum of the interior angles of a hexagon.
8. Find the exterior angle and the interior angle of a regular nonagon.
9. A regular polygon has an exterior angle of 30°. How many sides?
10. A regular polygon has an interior angle of 150°. How many sides?
11. State the number of lines of symmetry and the order of rotational symmetry of a rhombus.
12. Which condition do two triangles meet if all three pairs of sides are equal?
13. Do three equal angles (AAA) prove two triangles are congruent?
14. Two similar triangles have corresponding sides of 5 cm and 15 cm. What is the scale factor?
15. Corresponding sides of similar triangles are 6 cm and 9 cm. A side of the small one is 10 cm. Find the matching side.
16. EXT The length scale factor is 3. What are the area and volume scale factors?
17. EXT Two similar solids have volumes 8 cm^3 and 125 cm^3. Find the ratio of their lengths.
18. EXT The angle at the circumference is 55°. Find the angle at the centre on the same arc.
19. EXT What is the size of an angle in a semicircle?
20. EXT One angle of a cyclic quadrilateral is 78°. Find the opposite angle.
21. EXT A chord is 3 cm from the centre of a circle of radius 5 cm. Find the length of the chord.
22. EXT O is 17 cm from P and the radius is 8 cm. Find the length of the tangent from P.
23. EXT The angle between a tangent and a chord is 55°. What is the angle in the alternate segment?

EXAM-STYLE

24. AB is parallel to CD. Alternate angles are (3x + 10)° and (5x − 30)°. Find x and the size of the angles. [3]
25. Each interior angle of a regular polygon is 162°. Find the number of sides. [3]
26. The five interior angles of a pentagon are x, x + 10, x + 20, x + 30 and x + 40 degrees. Find x. [4]
27. ABC is an isosceles triangle with AB = AC and angle ABC = 70°. Find angle BAC and the exterior angle at C. [3]
28. In triangle ABC, DE is parallel to BC, D is on AB and E is on AC. AD = 6 cm, DB = 3 cm and DE = 8 cm. Find BC. [3]
29. Using ruler and compasses only, construct the perpendicular bisector of a line AB of length 8 cm. [3]
30. EXT Two similar vases are 12 cm and 18 cm tall. The small vase holds 400 cm^3 and has a surface area of 300 cm^2. Find the volume of the large vase and its surface area. [4]
31. EXT ABCD is a cyclic quadrilateral. Angle ABC = (3x + 12)° and angle ADC = (2x + 28)°. Find x and angle ABC. [3]
32. EXT PT is a tangent to a circle with centre O. Angle OPT = 28°. (a) Write down angle OTP and give a reason. (b) Find angle TOP. [3]
33. EXT AB is a diameter of a circle and C is on the circle. Angle CAB = 36°. (a) Find angle ACB. (b) Find angle ABC. (c) The tangent at A makes an angle with AC. Find it and give a reason. [5]
34. EXT A chord of length 16 cm is 6 cm from the centre of a circle. Find the radius. [3]

Show answers

1. 26
2. 145°
3. 108°
4. 360°
5. 65°
6. 72°
7. 720°
8. 40° and 140°
9. 12
10. 12
11. 2 lines, order 2
12. SSS
13. No, they are only similar
14. 3
15. 15 cm
16. 9 and 27
17. 2 : 5
18. 110°
19. 90°
20. 102°
21. 8 cm
22. 15 cm
23. 55°
24. alternate angles are equal
    3x + 10 = 5x − 30, so 2x = 40 and x = 20
    angle = 3(20) + 10 = 70°
25. exterior angle = 180 − 162 = 18°
    n = 360 ÷ 18
    n = 20
26. sum of interior angles of a pentagon = (5 − 2) × 180 = 540°
    5x + 100 = 540
    5x = 440, so x = 88
27. AC = AB, so angle ACB = 70°
    angle BAC = 180 − 70 − 70 = 40°
    exterior angle at C = 180 − 70 = 110° (= 40° + 70°)
28. triangles ADE and ABC are similar
    AB = 9 cm, so scale factor = 9 ÷ 6 = 1.5
    BC = 8 × 1.5 = 12 cm

29. draw AB = 8 cm
    with compasses set to more than 4 cm, draw arcs from A and from B so that they cross above and below AB
    join the two crossing points with a straight line

30. k = 18 ÷ 12 = 1.5
    volume = $400 \times 1.5^3 = 400 \times 3.375 = 1350$ cm³
    surface area = $300 \times 1.5^2 = 300 \times 2.25 = 675$ cm²

31. opposite angles add up to 180°
    3x + 12 + 2x + 28 = 180, so 5x = 140 and x = 28
    angle ABC = 3(28) + 12 = 96°

32. (a) OTP = 90° because a tangent is perpendicular to the radius at the point of contact
    (b) angles in a triangle: TOP = 180 − 90 − 28 = 62°

33. (a) angle in a semicircle: ACB = 90°
    (b) ABC = 180 − 90 − 36 = 54°
    (c) the angle between the tangent at A and chord AC = 54° (alternate segment theorem)

34. the perpendicular bisects the chord: half chord = 8 cm
    $r^2 = 8^2 + 6^2 = 64 + 36 = 100$
    r = 10 cm

Stuck on a question?

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Ask Riyaz Sir

Want regular help? Send this to your parent →