IGCSE MATHEMATICS 0580 · CHAPTER 8 NOTES · ~28 MIN READ

Mensuration — area & volume

Convert units, find area and perimeter, then master circles, arcs, sectors, prisms, cylinders, cones, spheres and frustums. Every formula is explained with a picture so you understand where it comes from.

Units | Perimeter & area | Circles, arcs & sectors | Surface area | Volume of prisms, cylinders, cones, spheres

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No tag = **Core** (everyone) EXTENDED Extended only highlight = comes up again and again

Test yourself at the end

IN 30 SECONDS

- **Area** is measured in **squared** units and **volume** in **cubed** units. To convert, use the length factor, **squared** for area and **cubed** for volume.
- **Circle:** C = **2πr** = **πd** and A = **πr²**. For **arcs and sectors** take a **fraction θ ÷ 360** of the whole circle. **Prism:** V = **area of cross-section × length**.
- **Surface area = the area of the net.** Cones, pyramids and spheres (Extended) have formulae that are given on the **formula sheet**: you only need to **use** them.

BY THE END YOU CAN

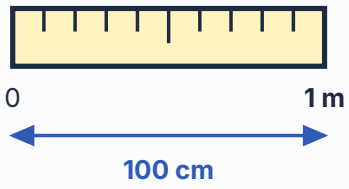
- Convert between units of length, area, volume and capacity.
- Find the perimeter and area of rectangles, triangles, parallelograms, trapezia and compound shapes.
- Find the circumference and area of circles, and the arc length and area of sectors.
- Find the volume and surface area of cuboids, prisms and cylinders.
- Find the volume and surface area of pyramids, cones, spheres and hemispheres. EXT
- Work with frustums, composite solids and the area of a segment. EXT

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1 Units of measure

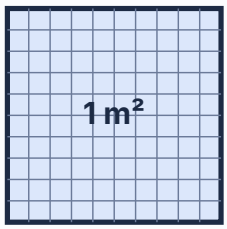
Length, area and volume

Length



1 m = 100 cm
1 cm = 10 mm
1 km = 1000 m

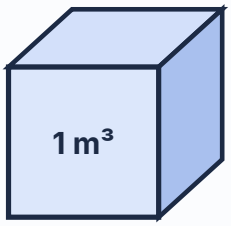
Area



100 cm × 100 cm

1 m² = 10 000 cm²
1 cm² = 100 mm²
1 km² = 1 000 000 m²

Volume



100 × 100 × 100 cm

1 m³ = 1 000 000 cm³
1 litre = 1000 cm³
1 m³ = 1000 litres

Area: square the length factor. Volume: cube it.

A metre is 100 cm, but a square metre is 100 × 100 = 10 000 cm², and a cubic metre is 100 × 100 × 100 = 1 000 000 cm³.

Capacity — how much a container holds. **1 litre = 1000 cm³ = 1000 ml**, so **1 ml = 1 cm³**, and **1 m³ = 1000 litres**.

To change	Length	Area	Volume
m to cm	× 100	× 10 000	× 1 000 000
cm to mm	× 10	× 100	× 1000
km to m	× 1000	× 1 000 000	× 1 000 000 000

- **Going to a smaller unit** (m to cm) **multiply**. **Going to a bigger unit** (cm to m) **divide**.
- **Square the factor for area and cube it for volume.** This is the step most students forget.

WORKED EXAMPLE

Change 3.5 m² to cm².

1. 1 m = 100 cm, so 1 m² = 100² = 10 000 cm²
2. **3.5 × 10 000 = 35 000 cm²**

WORKED EXAMPLE

Change 2 400 000 cm³ to m³.

1. 1 m³ = 1 000 000 cm³
2. **2 400 000 ÷ 1 000 000 = 2.4 m³**

WORKED EXAMPLE

A tank measures 50 cm by 40 cm by 30 cm. How many litres of water does it hold when full?

1. volume = 50 × 40 × 30 = 60 000 cm³
2. 1 litre = 1000 cm³
3. **60 000 ÷ 1000 = 60 litres**

COMMON MISTAKE

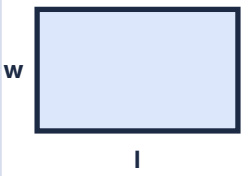
Using **1 m² = 100 cm²**. That is the length factor. Area needs **100² = 10 000**.

EXAM TIP

Always **convert to one unit first**, then calculate. Mixing metres and centimetres in one formula gives wrong answers.

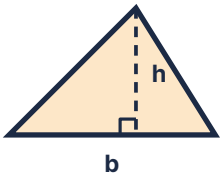
2 Perimeter and area

Four shapes to know



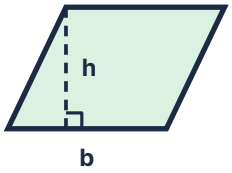
Rectangle

A = l × w
P = 2(l + w)



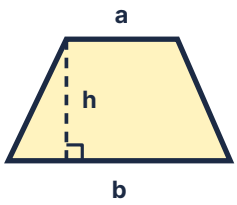
Triangle

A = ½ × b × h
h = perpendicular height



Parallelogram

A = b × h
h = upright height (not the slant)



Trapezium

A = ½(a + b) × h
a and b are the parallel sides

Area is measured with the perpendicular (upright) height. Perimeter is the distance all the way round.

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Perimeter — the **distance around** the edge of a shape. **Area** is the **space inside** it.

- **Use the perpendicular height** (at right angles to the base) in the triangle, parallelogram and trapezium formulae. **Never use the slanted side.**
- **Rearranging:** to find a missing length, use the formula backwards. For a triangle, h = 2A ÷ b.

WORKED EXAMPLE

Find the area and perimeter of a rectangle 12 cm by 7 cm.

1. area = 12 × 7 = **84 cm²**
2. **perimeter = 2(12 + 7) = 38 cm**

WORKED EXAMPLE

Find the area and perimeter of a right-angled triangle with legs 6 cm and 8 cm.

1. area = ½ × 6 × 8 = **24 cm²**
2. hypotenuse = √(6² + 8²) = √100 = 10 cm
3. **perimeter = 6 + 8 + 10 = 24 cm**

WORKED EXAMPLE

A parallelogram has base 10 cm, slanted side 6 cm and perpendicular height 5 cm. Find its area and perimeter.

1. area = base × perpendicular height = 10 × 5 = **50 cm²** (the 6 cm is not used)
2. **perimeter = 2(10 + 6) = 32 cm**

WORKED EXAMPLE

Find the area of a trapezium with parallel sides 7 cm and 11 cm and height 5 cm.

1. area = ½(7 + 11) × 5
2. **= ½ × 18 × 5 = 45 cm²**

WORKED EXAMPLE

A triangle has area 36 cm² and base 9 cm. Find its height.

1. ½ × 9 × h = 36
2. 4.5h = 36
3. **h = 8 cm**

WORKED EXAMPLE

A trapezium has area 60 cm² and parallel sides 8 cm and 12 cm. Find its height.

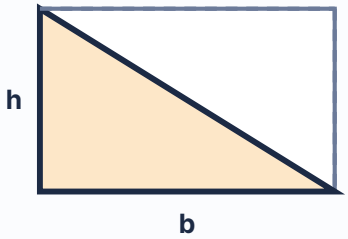
1. ½(8 + 12) × h = 60
2. 10h = 60
3. **h = 6 cm**

COMMON MISTAKE

Using the **slanted side** as the height of a parallelogram or triangle. If the question gives a slanted side, check that you are using the **upright** one.

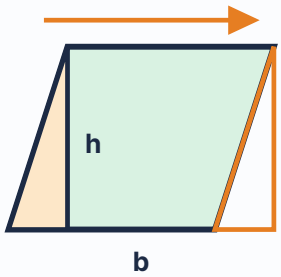
Why the formulae work

Triangle



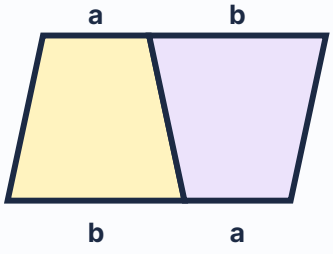
triangle = half of a rectangle
A = ½ × b × h

Parallelogram



move the cut triangle: parallelogram = rectangle
A = b × h

Trapezium



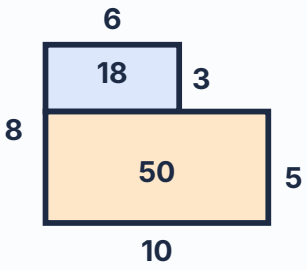
two copies make a parallelogram: base a + b
A = ½(a + b) × h

Each formula comes from a rectangle: cut it, slide it or double it.

- **Triangle:** a rectangle cut along a diagonal gives **two** equal triangles, so the triangle is **half** the rectangle.
- **Parallelogram:** move the triangle on one side to the other side and you get a **rectangle** with the same base and height.
- **Trapezium:** two copies make a **parallelogram** with base **(a + b)**, so one trapezium is half of it.

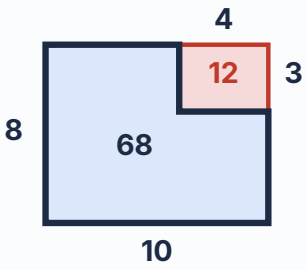
Compound shapes

Add the pieces



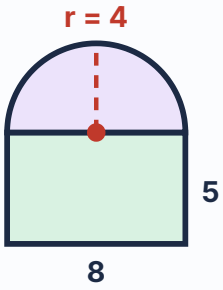
18 + 50 = 68 cm²
split into rectangles

Subtract the hole



80 − 12 = 68 cm²
subtract the hole

Semicircle on top



40 + ½π(4)² = 65.1 cm²
rectangle + semicircle

Split the shape into pieces you know, add them or subtract the part that is missing.

- **Method 1:** split into rectangles, triangles and circle parts, find each area and **add**.
- **Method 2:** find the area of the **big shape** and **subtract** the part that is not there (a hole or a corner).
- **Perimeter:** add up only the **outside edges**. Where two shapes join, the joining line is **not** part of the perimeter.

WORKED EXAMPLE

Find the area of an L-shape that fits in a 10 cm by 8 cm rectangle with a 4 cm by 3 cm corner removed.

1. big rectangle = 10 × 8 = 80 cm²
2. missing corner = 4 × 3 = 12 cm²
3. area = 80 − 12 = **68 cm²**
4. **check by adding: 6 × 3 + 10 × 5 = 18 + 50 = 68 ✓**

WORKED EXAMPLE

A shape is a rectangle 8 cm by 5 cm with a semicircle of radius 4 cm on one of its 8 cm sides. Find its area and perimeter.

1. area = 8 × 5 + ½ × π × 4² = 40 + 25.13 = **65.1 cm²**
2. arc length = ½ × 2π × 4 = 4π = 12.57 cm
3. **perimeter = 8 + 5 + 5 + 12.57 = 30.6 cm (the 8 cm where the semicircle joins is not counted twice)**

WORKED EXAMPLE

A square of side 10 cm has a circle of radius 4 cm cut out of it. Find the area that is left.

1. square = 10 × 10 = 100 cm²
2. circle = π × 4² = 16π = 50.27 cm²
3. **left = 100 − 50.27 = 49.7 cm²**

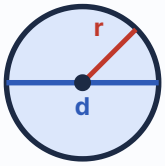
COMMON MISTAKE

Including the **joining line** in the perimeter of a compound shape, or **forgetting a curved edge**. Trace the outside with a pencil once.

3 Circles, arcs and sectors

Circumference and area

Circumference C = π × d

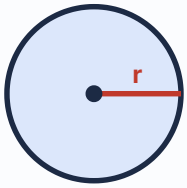


wrap the circle onto a line:



3 diameters and a bit: C ≈ 3.14 × d
also C = 2πr (d = 2r)

Area A = π × r²



fill it with squares of side r:



3 squares and a bit: A ≈ 3.14 × r²
use the π button on your calculator

A circle's circumference is a little more than 3 diameters. Its area is a little more than 3 squares of side r.

π (pi) — the number **3.14159...** that you get when you divide the circumference by the diameter of **any** circle. Use the **π button** on your calculator.

$$\text{circumference } C = \pi d = 2\pi r$$

d = diameter, r = radius. d = 2r

$$\text{area } A = \pi r^2$$

square the radius first, then multiply by π

- **Leave in terms of π** when the question asks (for example 16π cm²). Otherwise give **3 significant figures** from the calculator value of π.
- **Half circle (semicircle):** perimeter = πr + 2r (the curved edge **plus the diameter**). Area = ½πr².

WORKED EXAMPLE

Find the circumference and area of a circle with radius 7 cm.

1. C = 2π × 7 = 14π = **44.0 cm**
2. **A = π × 7² = 49π = 154 cm²**

WORKED EXAMPLE

Find the circumference and area of a circle with diameter 12 cm.

1. radius = 12 ÷ 2 = 6 cm
2. C = π × 12 = **37.7 cm**
3. **A = π × 6² = 36π = 113 cm²**

WORKED EXAMPLE

The circumference of a circle is 30 cm. Find its radius and area.

1. 2πr = 30, so r = 30 ÷ (2π) = **4.77 cm**
2. **A = π × 4.775² = 71.6 cm² (use the unrounded value of r on your calculator)**

WORKED EXAMPLE

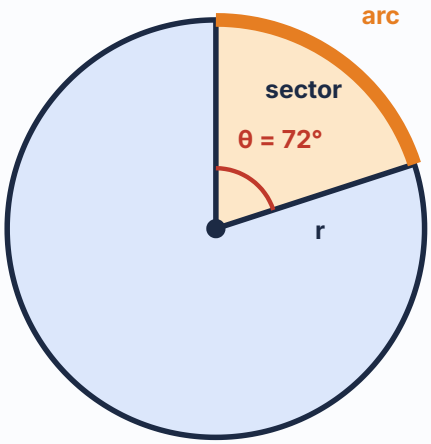
A circular lawn has area 100 m². Find its radius.

1. πr² = 100
2. r² = 100 ÷ π = 31.83
3. **r = √31.83 = 5.64 m**

COMMON MISTAKE

Putting the **diameter into πr²**. If a question gives the diameter, **halve it first**.

Arc length and sector area



Arc and sector

fraction = θ ÷ 360
arc length = fraction × 2πr
sector area = fraction × πr²
example: r = 10 cm, θ = 72°
fraction = 72 ÷ 360 = 1/5
arc = 1/5 × 2π × 10 = 12.6 cm
area = 1/5 × π × 10² = 62.8 cm²

An arc and a sector are fractions of the circle. The fraction is the angle at the centre divided by 360°.

Arc — part of the circumference of a circle.

Sector — the slice of the circle between two radii and an arc.

$$\text{arc length} = \frac{\theta}{360} \times 2\pi r$$

θ = angle at the centre, in degrees

$$\text{sector area} = \frac{\theta}{360} \times \pi r^2$$

the same fraction of the area

- **Step 1:** write the **fraction** θ ÷ 360 (simplify it). **Step 2:** multiply it by the **circumference** or the **area** of the whole circle.
- **Perimeter of a sector** = arc length + 2 × radius (do not forget the two straight edges).

WORKED EXAMPLE

A sector has radius 6 cm and angle 150°. Find the arc length, area and perimeter.

1. fraction = 150 ÷ 360 = $\frac{5}{12}$
2. arc = $\frac{5}{12}$ × 2π × 6 = 5π = **15.7 cm**
3. area = $\frac{5}{12}$ × π × 6² = 15π = **47.1 cm²**
4. **perimeter = 15.7 + 2 × 6 = 27.7 cm**

WORKED EXAMPLE

An arc of length 8.4 cm lies on a circle of radius 6 cm. Find the angle at the centre.

1. circumference = 2π × 6 = 37.70 cm
2. fraction = 8.4 ÷ 37.70 = 0.2228
3. **θ = 0.2228 × 360 = 80.2°**

WORKED EXAMPLE

A sector of a circle of radius 8 cm has area 40 cm². Find its angle.

1. whole circle = π × 8² = 201.06 cm²
2. fraction = 40 ÷ 201.06
3. **θ = 40 ÷ 201.06 × 360 = 71.6°**

COMMON MISTAKE

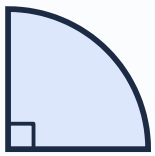
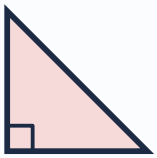
Using the **fraction upside down** or using **360 ÷ θ**. The fraction must be **less than 1**: θ on top, 360 on the bottom.

MEMORY TRICK

"Fraction × whole": a sector is a fraction of the **whole circle**. If the angle is 90° you are taking a **quarter** of the circle.

Segments

EXTENDED

Sector		Triangle		Segment
	−		=	
¼ × π × 10² = 78.5 cm²		**½ × 10 × 10 = 50 cm²**		**78.5 − 50 = 28.5 cm²**

segment = sector − triangle (radius 10 cm, angle 90°)

A segment is what is left when you cut the triangle away from the sector.

Segment — the region between a **chord** and an **arc**. It looks like a slice of a lens.

- EXT **Area of segment = area of sector − area of triangle.** The triangle has two sides equal to the radius, so its area is **½r² sin θ**.
- EXT For θ = 90° the triangle is right-angled and its area is just ½ × r × r.

WORKED EXAMPLE

EXT **Find the area of the segment cut off by a chord that makes an angle of 90° at the centre of a circle of radius 10 cm.**

1. sector = ¼ × π × 10² = 78.54 cm²
2. triangle = ½ × 10 × 10 = 50 cm²
3. **segment = 78.54 − 50 = 28.5 cm²**

WORKED EXAMPLE

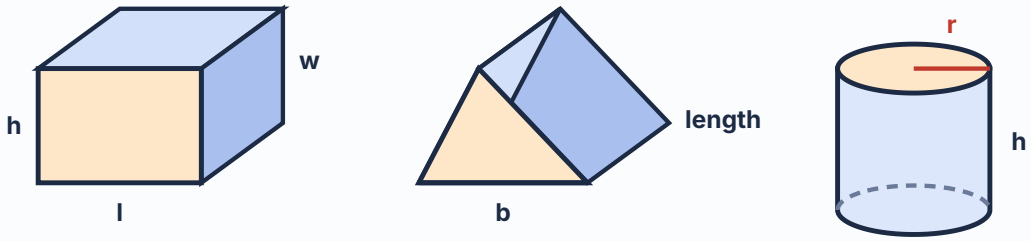
EXT **A chord makes an angle of 60° at the centre of a circle of radius 8 cm. Find the area of the segment.**

1. sector = $\frac{60}{360}$ × π × 8² = 33.51 cm²
2. triangle = ½ × 8 × 8 × sin 60° = 27.71 cm²
3. **segment = 33.51 − 27.71 = 5.80 cm²**

4 Prisms, cylinders and surface area

Volume of a prism

Volume of a prism = area of cross-section × length



cuboid
V = l × w × h

triangular prism
V = ½bh × length

cylinder
V = πr² × h

the shaded face is the cross-section

A prism has the same cross-section all the way along its length, so volume = area of cross-section × length.

Prism — a solid with the **same cross-section** throughout, such as a cuboid, a triangular prism or a cylinder.

volume of a prism = area of cross-section × length

works for every prism and the cylinder

cylinder: $V = \pi r^2 h$

the cross-section is a circle

- **Step 1:** find the **area of the cross-section** (the shape you see if you slice the prism). **Step 2:** multiply by the **length**.

WORKED EXAMPLE

Find the volume of a cuboid 8 cm by 5 cm by 3 cm.

1. **V = 8 × 5 × 3 = 120 cm³**

WORKED EXAMPLE

A prism has a triangular cross-section with base 6 cm and height 4 cm. The prism is 10 cm long. Find its volume.

1. cross-section area = ½ × 6 × 4 = 12 cm²
2. **V = 12 × 10 = 120 cm³**

WORKED EXAMPLE

Find the volume of a cylinder with radius 5 cm and height 12 cm.

1. V = π × 5² × 12 = 300π
2. **= 942 cm³**

WORKED EXAMPLE

A cylinder has volume 500 cm³ and height 8 cm. Find its radius.

1. πr² × 8 = 500
2. r² = 500 ÷ (8π) = 19.89
3. **r = 4.46 cm**

WORKED EXAMPLE

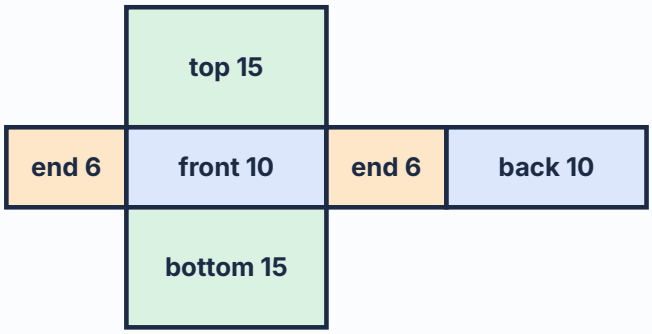
A cylindrical tank has radius 0.5 m and height 1.2 m. How many litres does it hold?

1. V = π × 0.5² × 1.2 = 0.3π = 0.9425 m³
2. 1 m³ = 1000 litres
3. **942 litres**

COMMON MISTAKE

Mixing **cm and m** in one calculation. Convert **everything to the same unit** before you multiply.

Surface area of cuboids and prisms



Cuboid 5 cm × 3 cm × 2 cm

top + bottom: 2 × 15 = 30
front + back: 2 × 10 = 20
two ends: 2 × 6 = 12
total = 62 cm²
2(lw + lh + wh)
= 2(15 + 10 + 6) = 62

Surface area = the area of the net. Add every face.

Surface area is the total area of every face. Draw the net and tick off each face.

- **Surface area = the area of the net.** Count **every** face, including the ones you cannot see.
- **Cuboid:** surface area = 2(lw + lh + wh).

WORKED EXAMPLE

Find the surface area of a cuboid 5 cm by 3 cm by 2 cm.

1. top and bottom: 2 × (5 × 3) = 30
2. front and back: 2 × (5 × 2) = 20
3. two ends: 2 × (3 × 2) = 12
4. **total = 62 cm²**

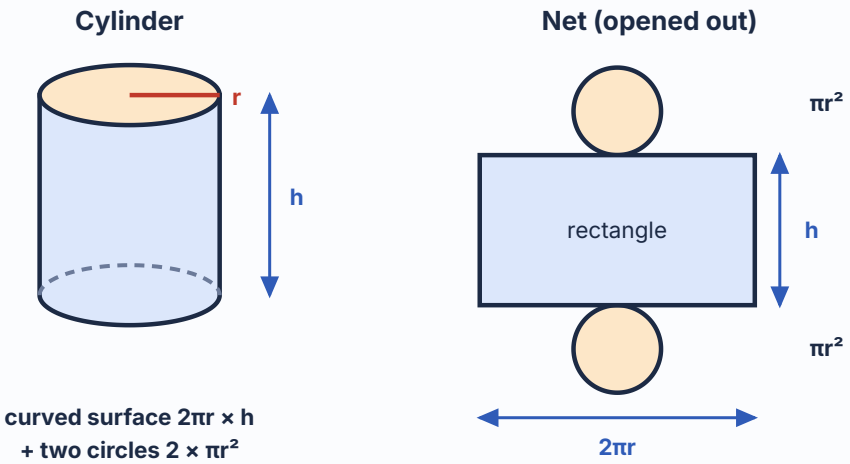
WORKED EXAMPLE

A triangular prism has a right-angled triangle cross-section with sides 3 cm, 4 cm and 5 cm and length 10 cm. Find its surface area.

1. two triangles: 2 × (½ × 3 × 4) = 12
2. three rectangles: (3 + 4 + 5) × 10 = 120
3. **total = 132 cm²**

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Surface area of a cylinder



curved surface $2\pi r \times h$ + two circles $2 \times \pi r^2$

total surface area $= 2\pi rh + 2\pi r^2$

Open a cylinder out: the curved part is a rectangle as wide as the circumference of the circle.

- **Cylinder:** curved surface = $2\pi rh$ and the **two circular ends** = $2 \times \pi r^2$. Total = **$2\pi rh + 2\pi r^2$**.

WORKED EXAMPLE

Find the total surface area of a cylinder with radius 5 cm and height 12 cm.

1. curved surface $= 2\pi \times 5 \times 12 = 120\pi$
2. two ends $= 2 \times \pi \times 5^2 = 50\pi$
3. **total $= 170\pi = 534$ cm²**

WORKED EXAMPLE

An open cylinder (no top) has radius 4 cm and height 10 cm. Find its outside surface area.

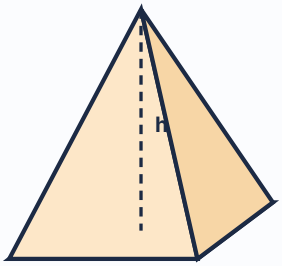
1. curved surface $= 2\pi \times 4 \times 10 = 80\pi$
2. one base $= \pi \times 4^2 = 16\pi$
3. **total $= 96\pi = 302$ cm²**

Riyaz Sir

5 Pyramids, cones and spheres EXT

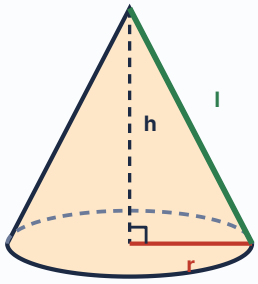
Pyramids and cones

Pyramid



V = ⅓ × base area × h
a third of the prism
with the same base and height

Cone



V = ⅓πr²h
curved surface = πrl
l² = r² + h² (slant height)

A pyramid or cone is exactly one third of the prism or cylinder with the same base and height.

pyramid: $V = \frac{1}{3} \times \text{base area} \times h$

h = perpendicular height

cone: $V = \frac{1}{3}\pi r^2 h$

curved surface area = πrl

slant height $l^2 = r^2 + h^2$

Pythagoras in the right-angled triangle inside the cone

- EXT **Total surface area of a cone** = curved surface + circular base = **πrl + πr²**.
- EXT The **slant height** l is the length **along the slope**. The **vertical height** h is **inside** the cone. Do not mix them up.

WORKED EXAMPLE

EXT **A square-based pyramid has base 6 cm by 6 cm and height 7 cm. Find its volume.**

1. base area = 6 × 6 = 36 cm²
2. **V = ⅓ × 36 × 7 = 84 cm³**

WORKED EXAMPLE

EXT **A cone has radius 5 cm and vertical height 12 cm. Find its slant height, volume and total surface area.**

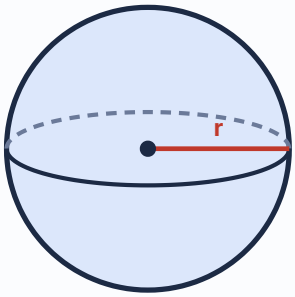
1. $l^2 = 5^2 + 12^2 = 169$, so **l = 13 cm**
2. $V = \frac{1}{3} \times \pi \times 5^2 \times 12 = 100\pi$ = **314 cm³**
3. curved surface = π × 5 × 13 = 65π
4. base = π × 5² = 25π
5. **total = 90π = 283 cm²**

COMMON MISTAKE

EXT Using the **slant height in the volume formula**. Volume needs the **vertical height h**. Slant height only appears in the **curved surface area**.

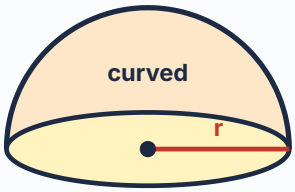
Spheres and hemispheres

Sphere



V = 4/3 πr³
surface area = 4πr²

Hemisphere



flat circle

V = 2/3 πr³
curved surface = 2πr²
with the flat base = 3πr²

The formulae for the sphere, cone and pyramid are on the formula sheet.

A sphere is described by one number, its radius. The formulae are on the formula sheet.

sphere: $V = 4/3\pi r^3$

surface area = $4\pi r^2$

hemisphere: $V = \frac{2}{3}\pi r^3$

curved surface = $2\pi r^2$

- EXT A **hemisphere** is half a sphere. Its **total surface area** adds the **flat circle**: $2\pi r^2 + \pi r^2 = \mathbf{3\pi r^2}$.

WORKED EXAMPLE

EXT **Find the volume and surface area of a sphere of radius 6 cm.**

1. $V = 4/3 \times \pi \times 6^3 = 288\pi =$ **905 cm³**
2. **surface area = $4\pi \times 6^2 = 144\pi = 452$ cm²**

WORKED EXAMPLE

EXT **Find the volume and total surface area of a solid hemisphere of radius 3 cm.**

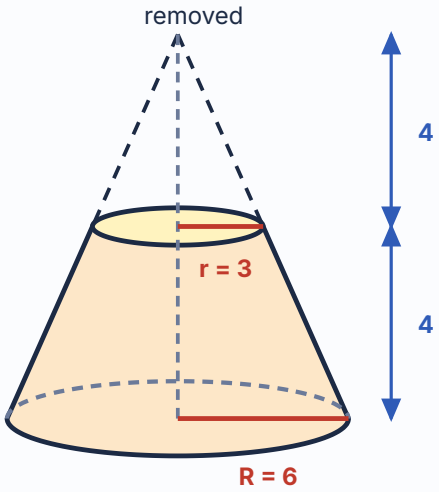
1. $V = \frac{2}{3} \times \pi \times 3^3 = 18\pi =$ **56.5 cm³**
2. curved surface $= 2\pi \times 3^2 = 18\pi$
3. flat base $= \pi \times 3^2 = 9\pi$
4. **total = $27\pi = 84.8$ cm²**

WORKED EXAMPLE

EXT **A sphere has volume 500 cm³. Find its radius.**

1. $4/3 \times \pi \times r^3 = 500$
2. $r^3 = 500 \times 3 \div (4\pi) = 119.37$
3. **$r = \sqrt[3]{119.37} = 4.92$ cm**

Frustums



Frustum = big cone − small cone

similar cones: r ÷ R = h ÷ H
3 ÷ 6 = 4 ÷ 8
big cone: ⅓π(6²)(8) = 96π
small cone: ⅓π(3²)(4) = 12π
frustum = 96π − 12π
= 84π = 264 cm³

A frustum is a cone with its top cut off. Find its volume by subtracting the small cone from the big one.

Frustum — the part of a cone left when a smaller cone is cut off the top by a plane **parallel to the base**.

- EXT **Frustum = big cone − small cone.** The small cone is **similar** to the big one, so r ÷ R = h ÷ H.
- EXT Use the similar-cones ratio to find the **missing height**, then subtract the two volumes.

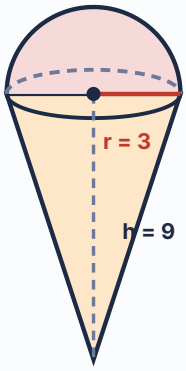
WORKED EXAMPLE

EXT **A cone of height 8 cm and base radius 6 cm has a small cone of height 4 cm and radius 3 cm removed from the top. Find the volume of the frustum.**

1. big cone = ⅓ × π × 6² × 8 = 96π
2. small cone = ⅓ × π × 3² × 4 = 12π
3. **frustum = 96π − 12π = 84π = 264 cm³**

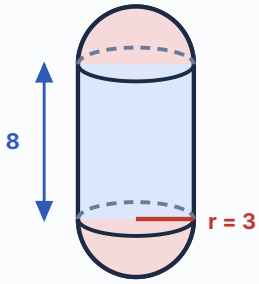
Composite solids

Cone + hemisphere



cone: ⅓π(3²)(9) = 27π
hemisphere: ⅔π(3³) = 18π
total = 45π = 141 cm³

Capsule: cylinder + 2 hemispheres



cylinder: π(3²)(8) = 72π
2 hemispheres: 4/3π(3³) = 36π
total = 108π = 339 cm³

Split the solid into simple solids, find each volume, then add them.

- EXT **Hidden faces:** when you join two solids, the joined faces are **not** part of the surface area.

WORKED EXAMPLE

EXT An ice cream is a cone of radius 3 cm and height 9 cm with a hemisphere on top of the same radius. Find its volume.

1. cone = ⅓ × π × 3^2 × 9 = 27π
2. hemisphere = ⅔ × π × 3^3 = 18π
3. **total = 45π = 141 cm^3**

WORKED EXAMPLE

EXT A capsule is a cylinder of radius 3 cm and length 8 cm with a hemisphere on each end. Find its volume.

1. cylinder = π × 3^2 × 8 = 72π
2. two hemispheres make one sphere: 4/3 × π × 3^3 = 36π
3. **total = 108π = 339 cm^3**

Problems with a context

- **Costs and amounts:** find the area or volume first, then divide by what one unit covers (and **round up** for whole boxes or tiles).

WORKED EXAMPLE

A rectangular lawn is 12 m by 9 m with a circular pond of radius 2 m. One box of seed covers 30 m^2. How many boxes are needed?

1. lawn area = 12 × 9 = 108 m^2
2. pond = π × 2^2 = 12.57 m^2
3. grass = 108 − 12.57 = 95.43 m^2
4. 95.43 ÷ 30 = 3.18
5. **you need 4 boxes (round up)**

WORKED EXAMPLE

A floor is 4.2 m by 3.5 m. Square tiles of side 30 cm are used. How many tiles are needed?

1. floor = 4.2 × 3.5 = 14.7 m^2
2. one tile = 0.3 × 0.3 = 0.09 m^2
3. 14.7 ÷ 0.09 = 163.3
4. **164 tiles**

Formula quick card

- Learn the Core formulae. The Extended solids are given on the formula sheet. Work from **this card** when you check.

Solid	Volume	Surface area
Cuboid	l × w × h	2(lw + lh + wh)
Any prism	cross-section × length	sum of the faces
Cylinder	$\pi r^2 h$	$2\pi rh + 2\pi r^2$
Cone (Ext)	$\frac{1}{3}\pi r^2 h$	$\pi rl + \pi r^2$
Pyramid (Ext)	⅓ × base area × h	sum of the faces
Sphere (Ext)	4/3 πr^3	$4\pi r^2$
Hemisphere (Ext)	$\frac{2}{3}\pi r^3$	$3\pi r^2$ with the flat base

Riyaz Sir

Test yourself — cover the answers first

QUICK CHECK

1. Change 4.5 m^2 to cm^2.
2. Change 3 500 000 cm^3 to m^3.
3. How many litres are in 2.5 m^3?
4. Find the area of a triangle with base 14 cm and height 9 cm.
5. Find the area of a trapezium with parallel sides 5 cm and 9 cm and height 4 cm.
6. Find the perimeter of a rectangle 9.5 cm by 4 cm.
7. Find the circumference of a circle with radius 5 cm, in terms of π.
8. Find the area of a circle with diameter 10 cm, in terms of π.
9. Find the area of a circle of radius 3.5 cm to 3 significant figures.
10. Find the circumference of a circle of diameter 8.2 cm to 3 significant figures.
11. A circle has circumference 44 cm. Find its radius to 3 significant figures.
12. Find the length of an arc of a circle with radius 9 cm and angle 40°.
13. Find the area of a sector with radius 12 cm and angle 90° to 3 significant figures.
14. Find the volume of a cuboid 6 cm by 4 cm by 2.5 cm.
15. Find the volume of a cylinder with radius 3 cm and height 10 cm in terms of π.
16. A prism has a cross-section of area 12 cm^2 and length 7 cm. Find its volume.
17. Find the surface area of a cube with side 5 cm.
18. EXT Find the volume of a cone with radius 3 cm and height 7 cm to 3 significant figures.
19. EXT Find the volume of a sphere of radius 2 cm in terms of π.
20. EXT Find the surface area of a sphere of radius 10 cm in terms of π.
21. EXT A cone has radius 6 cm and height 8 cm. Find its slant height.
22. EXT Find the volume of a square-based pyramid with base 5 cm by 5 cm and height 9 cm.
23. EXT Find the total surface area of a solid hemisphere of radius 6 cm in terms of π.

EXAM-STYLE

24. A trapezium has parallel sides 9 cm and 15 cm and an area of 84 cm^2. Find its height. [3]
25. A rectangle is 12 cm by 8 cm. A circle of radius 3 cm is cut out of it. Find the area of the shape that is left. [4]
26. A sector has radius 9 cm and angle 140°. Find (a) the arc length, (b) the area of the sector. [4]
27. A cylindrical tank has radius 0.8 m and height 2 m. Find its capacity in litres, to the nearest litre. [3]
28. A box is 40 cm by 25 cm by 30 cm. Find its surface area and its capacity in litres. [4]
29. A prism has a right-angled triangle cross-section with sides 5 cm, 12 cm and 13 cm. The prism is 20 cm long. Find its volume and its total surface area. [5]
30. EXT A cone has base radius 7 cm and slant height 25 cm. Find its vertical height, its volume and its curved surface area. Give answers to 3 significant figures. [5]
31. EXT A sphere has volume 36π cm^3. Find its radius and its surface area. [4]
32. EXT A solid is a cylinder of radius 4 cm and height 10 cm with a hemisphere of radius 4 cm on top. Find its volume to 3 significant figures. [4]
33. EXT A frustum is made by removing a cone of height 6 cm and radius 3 cm from the top of a cone of height 12 cm and radius 6 cm. Find the volume of the frustum to 3 significant figures. [4]

Show answers

1. 45 000 cm²
2. 3.5 m³
3. 2500
4. 63 cm²
5. 28 cm²
6. 27 cm
7. 10π cm
8. 25π cm²
9. 38.5 cm²
10. 25.8 cm
11. 7.00 cm
12. 6.28 cm (2π)
13. 113 cm²
14. 60 cm³
15. 90π cm³
16. 84 cm³
17. 150 cm²
18. 66.0 cm³
19. 32π ÷ 3 cm³ (33.5 cm³)
20. 400π cm²
21. 10 cm
22. 75 cm³
23. 108π cm²
24. area = ½(9 + 15) × h = 12h
    12h = 84
    h = 7 cm
25. rectangle = 12 × 8 = 96 cm²
    circle = π × 3² = 28.27 cm²
    left = 96 − 28.27 = 67.7 cm²
26. fraction = 140 ÷ 360 = 7/18
    (a) arc = 7/18 × 2π × 9 = 7π = 22.0 cm
    (b) area = 7/18 × π × 9² = 31.5π = 99.0 cm²
27. V = π × 0.8² × 2 = 1.28π = 4.0212 m³
    1 m³ = 1000 litres
    4021 litres
28. surface area = 2(40 × 25 + 40 × 30 + 25 × 30) = 2(1000 + 1200 + 750) = 5900 cm²
    volume = 40 × 25 × 30 = 30 000 cm³
    capacity = 30 000 ÷ 1000 = 30 litres

29. cross-section = ½ × 5 × 12 = 30 cm²
    volume = 30 × 20 = 600 cm³
    two triangles = 60 cm²
    rectangles = (5 + 12 + 13) × 20 = 600 cm²
    surface area = 660 cm²

30. h = √(25² − 7²) = √576 = 24 cm
    V = ⅓ × π × 7² × 24 = 392π = 1230 cm³
    curved surface = π × 7 × 25 = 175π = 550 cm²

31. 4/3πr³ = 36π, so r³ = 27
    r = 3 cm
    surface area = 4π × 3² = 36π = 113 cm²

32. cylinder = π × 4² × 10 = 160π
    hemisphere = ⅔ × π × 4³ = 128π ÷ 3
    total = 160π + 128π ÷ 3 = 608π ÷ 3 = 637 cm³

33. big cone = ⅓ × π × 6² × 12 = 144π
    small cone = ⅓ × π × 3² × 6 = 18π
    frustum = 144π − 18π = 126π = 396 cm³

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Share with a friend

Ask Riyaz Sir

Want regular help? Send this to your parent →