IGCSE MATHEMATICS 0580 · CHAPTER 2 NOTES · ~21 MIN READ

Ratio, percentages & money

Ratio, proportion, speed, percentages, interest and exchange rates: the everyday maths that fills IGCSE Paper 1 and Paper 3, taught with bar models, multipliers and full worked examples.

Ratio | Proportion | Rates & speed | Percentages | Reverse percentage | Interest | Growth & decay

RS by Riyaz Sir
mentorriyaz.in

No tag = **Core** (everyone) EXTENDED Extended only highlight = comes up again and again

Test yourself at the end

IN 30 SECONDS

- **Ratio** compares parts. Find the **value of one part** first (a bar model helps), then multiply. **Proportion** is the same idea: find the value of **1**, then scale.
- Every percentage question is a **multiplier**: increase 20% → × 1.2, decrease 15% → × 0.85. To go **backwards** (reverse percentage) you **divide** by the multiplier.
- **Compound interest** and **growth and decay** use **multiplier^n**. For speed use the triangle D, S, T, and keep the **units** matching.

BY THE END YOU CAN

- Simplify ratios, divide a quantity in a given ratio and solve ratio problems.
- Use the unitary method and scale factors for direct proportion, and find best buys.
- Calculate with speed, density and other rates, and convert between units of speed.
- Find percentages of amounts, percentage change, profit and loss, and use percentage multipliers.
- Solve reverse percentage problems.
- Calculate simple and compound interest.
- Work with time (12-hour and 24-hour clocks), timetables and currency conversion.
- Use direct and inverse proportion with x, x^2, x^3, $\sqrt{x}$, $1/x$ and $1/x^2$. EXT
- Solve exponential growth and decay problems. EXT

1 Ratio

Simplifying a ratio

Simplify: divide both parts by the HCF

12 : 18 → ÷ 6 → 2 : 3

simplest form:
no common factor left

Different units: convert first, then simplify

50 cm : 2 m → same units → 50 : 200 → ÷ 50 → 1 : 4

Write as 1 : n: divide both parts by the first

2 : 5 → ÷ 2 → 1 : 2.5

Divide both parts by the same number. For different units, convert first.

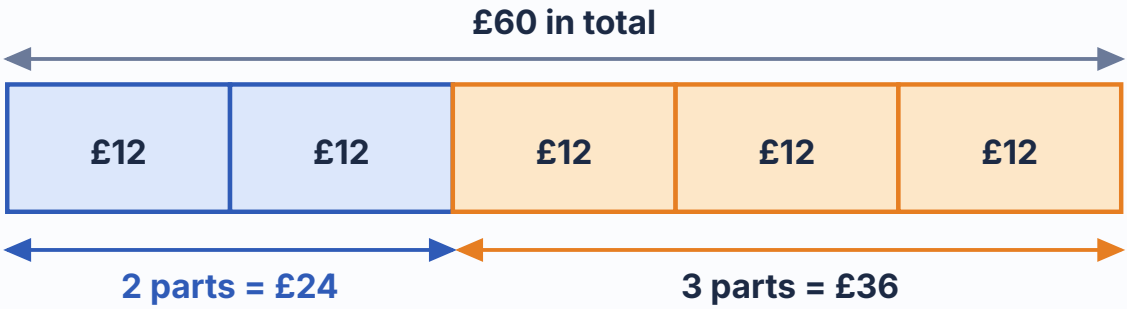
Ratio — a comparison of quantities in the same units, written a : b. Divide both parts by the **same number** and you get an **equivalent ratio**.

- **Simplest form:** divide by the **HCF** (see Number toolkit). 12 : 18 = 2 : 3.
- **Fractions or decimals:** multiply both sides to clear them. $\frac{2}{3} : \frac{1}{2} = 4 : 3$ (multiply by 6). 1.2 : 0.8 = 12 : 8 = 3 : 2.
- **1 : n form:** divide **both** by the first number. 2 : 5 = 1 : 2.5.
- A ratio has **no units** once simplified: 50 cm : 2 m = 50 : 200 = 1 : 4.

COMMON MISTAKE

Simplifying 50 cm : 2 m as 25 : 1 by **ignoring the units**. Convert to the **same unit** first.

Sharing in a ratio



1. total parts: 2 + 3 = 5
2. one part: £60 ÷ 5 = £12
3. shares: 2 × £12 = £24 and 3 × £12 = £36

Add the parts, find one part, then multiply. The shares must add back to the total.

- **Method:** (1) **add the parts**; (2) **divide** the total by this to find **one part**; (3) **multiply** each part by that number; (4) **check** that the shares add to the total.
- A ratio of 2 : 3 also means the shares are $\frac{2}{5}$ and $\frac{3}{5}$ of the total.

WORKED EXAMPLE

Share £480 in the ratio 3 : 5 : 4.

1. total parts = 3 + 5 + 4 = **12**
2. one part = 480 ÷ 12 = **£40**
3. shares: 3 × 40 = **£120**, 5 × 40 = **£200**, 4 × 40 = **£160**
4. **check: 120 + 200 + 160 = 480 ✓**

WORKED EXAMPLE

The ratio of boys to girls is 4 : 5. There are 36 boys. How many girls?

1. 4 parts = 36 boys
2. one part = 36 ÷ 4 = **9**
3. **girls = 5 parts = 5 × 9 = 45**

WORKED EXAMPLE

A and B share money in the ratio 3 : 7. B gets £48 more than A. How much is shared altogether?

1. difference in parts = 7 − 3 = **4 parts**
2. 4 parts = £48, so one part = **£12**
3. **total parts = 10, so total = 10 × 12 = £120**

EXAM TIP

When you are given **one share** (not the total), use **that share** to find one part. When you are given the **difference**, use the **difference in parts**.

2 Proportion

Unitary method and best buys

Unitary method: go to 1 first



Scale factor: when it divides nicely



scale factor
6 ÷ 4 = 1.5

Go to one first (divide), then to the amount you want (multiply). If the numbers divide nicely, use a scale factor.

Direct proportion — when one quantity **doubles**, the other **doubles** too. More pens cost more money.

- **Unitary method:** find the value of **one**, then multiply. **Scale factor:** multiply both quantities by the **same number**.
- **Best buy:** work out the **price per unit** (per gram, per litre) for each, then compare. The **lower price per unit** is the better value.

WORKED EXAMPLE

5 pens cost £3.50. Find the cost of 8 pens.

1. one pen = 3.50 ÷ 5 = **£0.70**
2. **8 pens = 8 × 0.70 = £5.60**

WORKED EXAMPLE

Which is the better buy: 500 g for £2.40 or 750 g for £3.45?

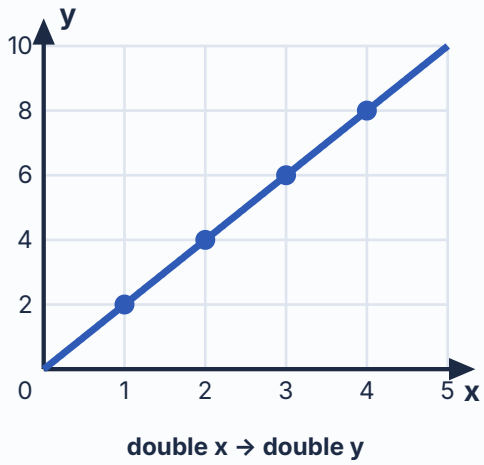
1. price per gram: 240p ÷ 500 = **0.48p**, 345p ÷ 750 = **0.46p**
2. the 750 g pack costs less per gram
3. **the 750 g pack is the better buy**

Direct and inverse proportion

EXTENDED

Direct proportion: y = kx

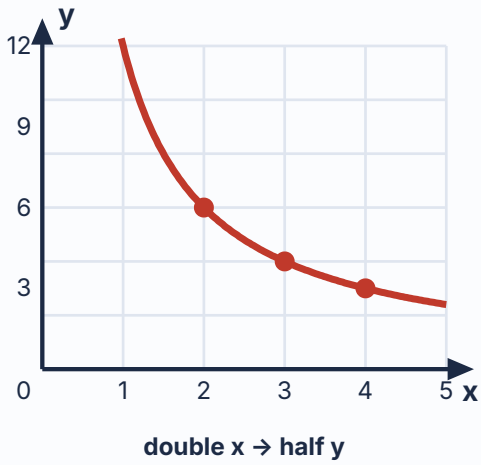
straight line through (0, 0)



double x → double y

Inverse proportion: y = k ÷ x

a curve: y falls as x rises



double x → half y

Direct proportion is a straight line through the origin. Inverse proportion is a curve that falls as x rises.

Riyaz Sir

Inverse proportion — EXT when one quantity **doubles**, the other **halves**. More workers means **fewer** days.

- EXT **Direct:** y = kx. The graph is a **straight line through (0, 0)**. **Inverse:** y = k ÷ x. The graph is a **curve** that never touches the axes.
- EXT **Method:** (1) write the **equation** with k; (2) use the **given pair** to find k; (3) rewrite the equation with that k; (4) substitute the new value.

WORKED EXAMPLE

EXT **y is directly proportional to x, and y = 12 when x = 4. Find y when x = 7.**

1. write y = kx
2. substitute: 12 = k × 4, so **k = 3**
3. equation: **y = 3x**
4. **when x = 7, y = 3 × 7 = 21**

WORKED EXAMPLE

EXT **y is inversely proportional to x, and y = 6 when x = 4. Find x when y = 3.**

1. write $y = \frac{k}{x}$
2. $6 = \frac{k}{4}$, so **k = 24**
3. equation: $y = \frac{24}{x}$
4. **when y = 3: $3 = \frac{24}{x}$, so x = 24 ÷ 3 = 8**

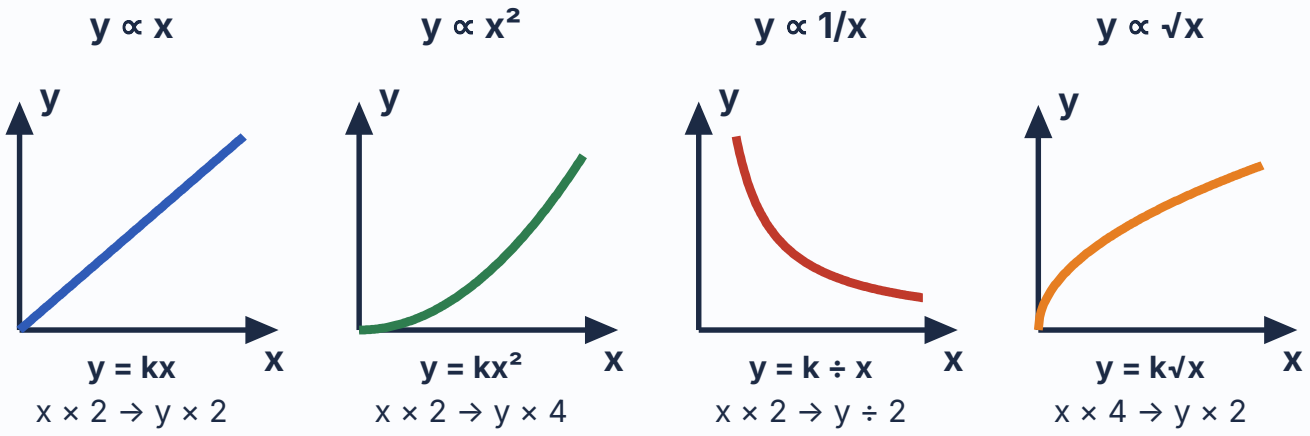
WORKED EXAMPLE

EXT **6 workers take 10 days to build a wall. How long would 4 workers take?**

1. more workers means fewer days: **inverse proportion**
2. total work = 6 × 10 = **60 worker-days**
3. **days = 60 ÷ 4 = 15 days**

Other forms of proportion

EXTENDED



Match the words to the equation. The graph shape tells you which form it is.

Statement	Equation	If x doubles
y is proportional to x	y = kx	y doubles
y is proportional to x^2	$y = kx^2$	y × 4
y is proportional to x^3	$y = kx^3$	y × 8
y is proportional to √x	y = k√x	y × √2
y is inversely proportional to x	y = k/x	y halves
y is inversely proportional to x^2	$y = k/x^2$	y ÷ 4

- EXT "**Proportional to**" always means a **constant k**. The same four steps work for every form: **find k first**.

WORKED EXAMPLE

EXT **y is proportional to x^2, and y = 50 when x = 5. Find y when x = 8.**

1. $y = kx^2$
2. $50 = k \times 25$, so **k = 2**
3. $y = 2x^2$
4. **when x = 8: $y = 2 \times 64 = 128$**

WORKED EXAMPLE

EXT **y is inversely proportional to the square of x. y = 4 when x = 3. Find y when x = 6.**

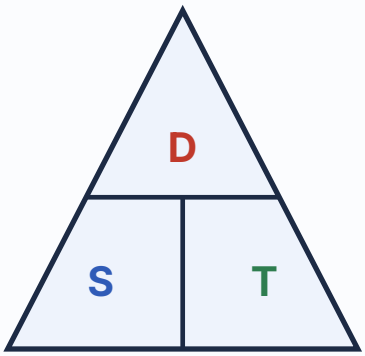
1. $y = \frac{k}{x^2}$
2. $4 = \frac{k}{9}$, so **k = 36**
3. $y = \frac{36}{x^2}$
4. **when x = 6: $y = \frac{36}{36} = 1$**

MEMORY TRICK

Direct → multiply, Inverse → divide. If the question says "**square of**", put the square on x, not on y.

3 Rates and speed

Speed, distance and time



cover the one you want

cover D	**D = S × T**
cover S	**S = D ÷ T**
cover T	**T = D ÷ S**

average speed = total distance ÷ total time

units must match: km, h, km/h or m, s, m/s

Cover the quantity you want to find. What is left shows the calculation.

Rate — a quantity **per unit of another**, for example km **per** hour, pence **per** gram, litres **per** minute.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

so Distance = Speed × Time and Time = Distance ÷ Speed

- **Units must match:** km, h and km/h **or** m, s and m/s. Convert the time first.
- **Minutes to hours:** divide by 60. 1 h 30 min = **1.5 h** (not 1.30). 2 h 15 min = **2.25 h**.
- **Average speed** = **total distance ÷ total time** for the whole journey. It is **not** the average of the speeds.
- **km/h to m/s:** multiply by 1000 and divide by 3600. So 72 km/h = 72 × 1000 ÷ 3600 = **20 m/s**. (Divide by 3.6.)

WORKED EXAMPLE

A car travels 150 km in 2 hours 30 minutes. Find its average speed.

1. change the time: 2 h 30 min = **2.5 h**
2. **speed = $\frac{150}{2.5}$ = 60 km/h**

WORKED EXAMPLE

A car drives 60 km at 40 km/h, then 60 km at 60 km/h. Find the average speed for the whole journey.

1. time for part 1 = 60 ÷ 40 = **1.5 h**
2. time for part 2 = 60 ÷ 60 = **1 h**
3. total distance = 120 km, total time = 2.5 h
4. **average speed = 120 ÷ 2.5 = 48 km/h (not 50)**

COMMON MISTAKE

Averaging the two speeds: (40 + 60) ÷ 2 = 50. The car spends **more time** at the slower speed, so the average is **below** 50.

Other rates

- **Density** = $\frac{\text{mass}}{\text{volume}}$, measured in g/cm³ or kg/m³. **Flow rate** = $\frac{\text{volume}}{\text{time}}$ (for example litres per minute).
- **Pro rata** means "in proportion": £7.50 for 3 hours is £2.50 per hour, so 5 hours costs £12.50.

WORKED EXAMPLE

A metal block has mass 540 g and volume 200 cm³. Find its density.

1. density = mass ÷ volume
2. = 540 ÷ 200
3. **= 2.7 g/cm³**

WORKED EXAMPLE

A tank holds 600 litres. A tap fills it at 25 litres per minute. How long does it take?

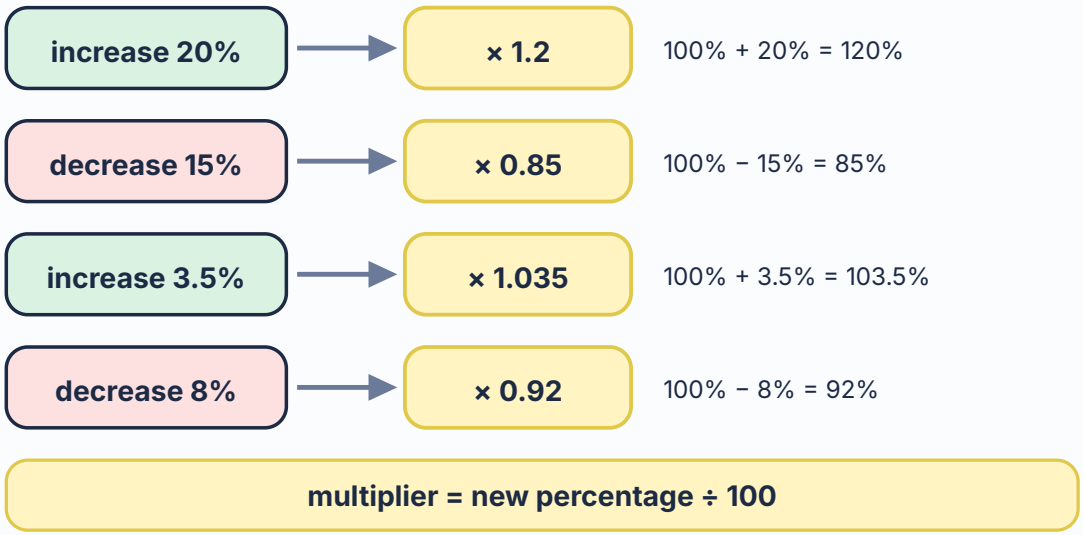
1. time = amount ÷ rate
2. = 600 ÷ 25
3. **= 24 minutes**

MEMORY TRICK

Draw the triangle for **any** rate: the **product** goes on top. Density has mass on top, and speed has distance on top. See also distance–time graphs.

4 Percentages

Percentage multipliers



Work out the new percentage first, then divide by 100. The multiplier does an increase or a decrease in one step.

Percentage — a fraction **out of 100**. 35% means $\frac{35}{100}$ = 0.35.

- **Percentage of an amount:** change to a decimal and multiply. 15% of 80 = 0.15 × 80 = **12**.
- **Express as a percentage:** write a fraction, then multiply by 100. 18 out of 72 = $\frac{18}{72}$ × 100 = **25%**.
- **Increase / decrease:** use a **multiplier**. Increase by 12% → × 1.12. Decrease by 35% → × 0.65.

WORKED EXAMPLE

Increase £250 by 12%.

1. new percentage = 100% + 12% = **112%**
2. multiplier = **1.12**
3. **250 × 1.12 = £280**

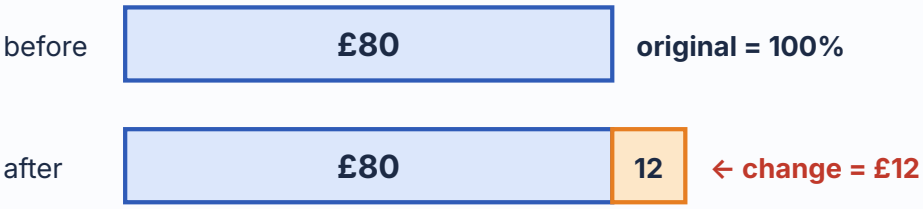
WORKED EXAMPLE

Decrease 640 by 35%.

1. new percentage = 100% − 35% = **65%**
2. multiplier = **0.65**
3. **640 × 0.65 = 416**

Percentage change and profit

A price rises from £80 to £92



percentage change = change ÷ original × 100

12 ÷ 80 × 100 = 15% increase

Divide by the ORIGINAL (the "before" amount), not the new one.

Always divide the change by the original value, which is the starting amount.

$$\% \text{ change} = \frac{\text{change}}{\text{original}} \times 100$$

change = new − old. Positive = increase, negative = decrease

- **Profit or loss:** % profit = $\frac{\text{profit}}{\text{cost price}} \times 100$. The **cost price** is the starting value.
- A **discount** or **VAT** is just a percentage change on the price.

WORKED EXAMPLE

A phone costs £450. In a sale it costs £369. Find the percentage decrease.

1. change = 450 − 369 = **£81**
2. $\frac{81}{450} \times 100$ = **18%**
3. **answer: 18% decrease**

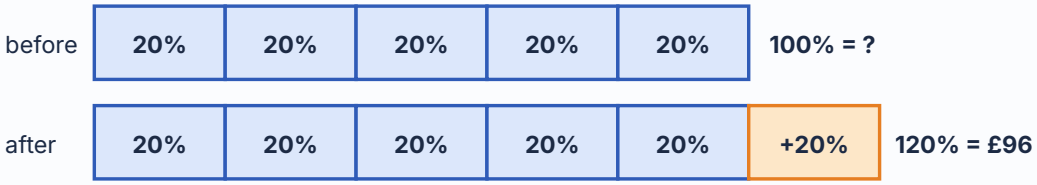
WORKED EXAMPLE

A trader buys a chair for £40 and sells it for £52. Find the percentage profit.

1. profit = 52 − 40 = **£12**
2. $\frac{12}{40} \times 100$ = **30%**
3. **divide by the cost price, not the selling price**

Reverse percentages

After a 20% rise the price is £96. Find the original.



1. 120% = £96

2. ÷ 6 → 20% = £16 (or ÷ 120 → 1% = £0.80)

3. × 5 → 100% = £80, the original price

Not 96 − 20% of 96: the 20% was added to the ORIGINAL.

The percentage was taken from the original, so divide by the multiplier to get back to 100%.

- **Method:** (1) write the **given amount** as a percentage of the **original**; (2) find **1%** (divide); (3) multiply by **100**. Or **divide by the multiplier**.

WORKED EXAMPLE

A jacket costs £51 after a 15% discount. Find the original price.

1. sale price = 100% − 15% = **85%** of the original
2. 85% = £51, so 1% = 51 ÷ 85 = £0.60
3. 100% = 0.60 × 100 = **£60**
4. **(check: 60 × 0.85 = 51 ✓)**

WORKED EXAMPLE

A price including 20% VAT is £90. Find the price before VAT.

1. price with VAT = **120%** of the original
2. original = 90 ÷ 1.2
3. **= £75**

COMMON MISTAKE

Working out 15% of the **sale price** and adding it back: 51 + 7.65 = 58.65. The 15% was taken from the **original**, not from £51, so **divide by 0.85** to get £60.

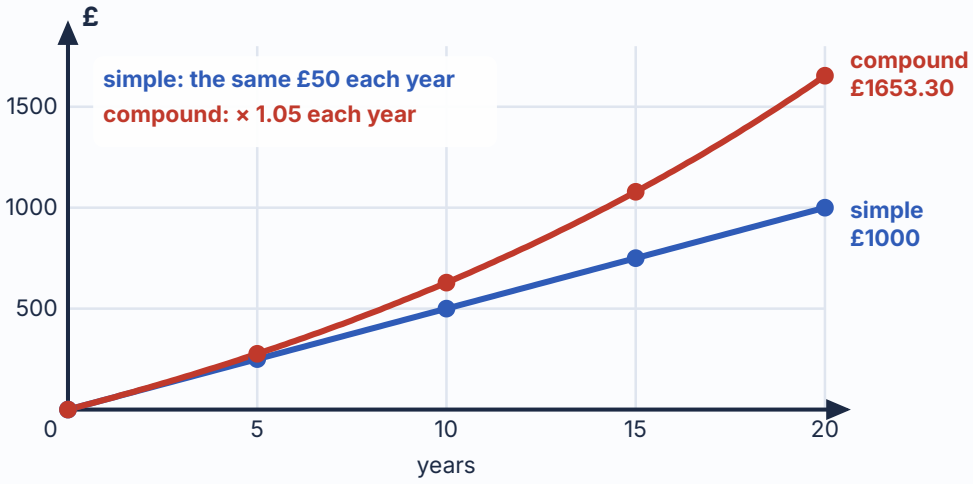
EXAM TIP

Check by **forward calculation**: multiply your answer by the multiplier. You should get the amount in the question.

5 Interest, growth and decay

Simple and compound interest

Interest earned on £1000 at 5% a year, over 20 years



Simple interest adds the same amount each year. Compound interest earns interest on the interest too, so it curves upwards.

Simple interest — interest on the **original amount only**, the same every year.

Compound interest — interest on the **original amount plus the interest already added**.

$$\text{Simple interest } I = \frac{P \times R \times T}{100}$$

P = principal, R = rate % per year, T = years

$$\text{Compound amount } A = P\left(1 + \frac{r}{100}\right)^n$$

n = number of years. The interest earned = A − P

WORKED EXAMPLE

Find the simple interest on £800 at 4.5% a year for 3 years.

1. $I = \frac{800 \times 4.5 \times 3}{100}$
2. = **£108**
3. **total amount = 800 + 108 = £908**

WORKED EXAMPLE

Find the compound amount when £2000 is invested at 3% a year for 4 years.

1. multiplier = $1 + \frac{3}{100}$ = **1.03**
2. $A = 2000 \times 1.03^4$
3. = **£2251.02**
4. **interest earned = 2251.02 − 2000 = £251.02**

WORKED EXAMPLE

£5000 is invested at 2% compound interest for 3 years. How much interest is earned?

1. $A = 5000 \times 1.02^3$
2. = **£5306.04**
3. **interest = 5306.04 − 5000 = £306.04**

EXAM TIP

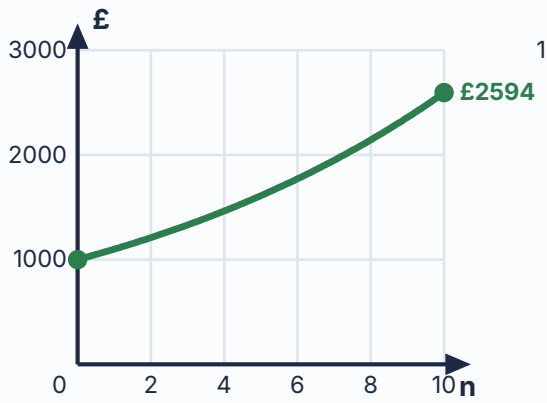
"Interest" and **"total amount"** are different. Read the last line of the question: do they want A or A − P?

Exponential growth and decay

EXTENDED

Growth: +10% a year

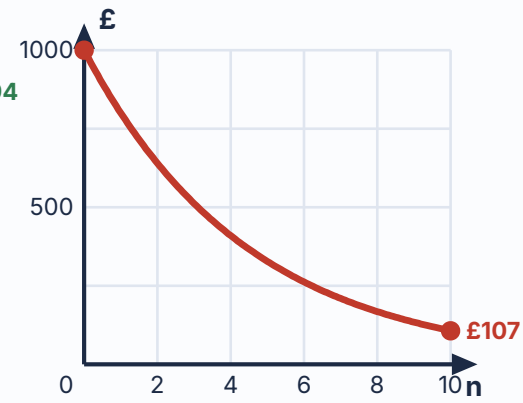
× 1.1 each year, $A = 1000 \times 1.1^n$



£1000 grows to £2594 in 10 years

Decay: −20% a year

× 0.8 each year, $A = 1000 \times 0.8^n$



never reaches 0, just gets smaller

Repeated multiplication gives a curve. A multiplier above 1 grows, and a multiplier between 0 and 1 decays.

$$\text{Final amount} = \text{initial} \times \text{multiplier}^n$$

multiplier = 1 + rate for growth, and 1 − rate for decay

- EXT **Growth:** +r% a year → multiplier $1 + \frac{r}{100}$ (population, investments). **Decay:** −r% a year → multiplier $1 - \frac{r}{100}$ (depreciation, radioactive decay).
- EXT This is the **same maths as compound interest**: only the context changes. Decay curves **never reach zero**.

WORKED EXAMPLE

EXT **A town has 12 000 people. The population grows by 2.5% a year. Find the population after 5 years.**

1. multiplier = **1.025**
2. $12\,000 \times 1.025^5$
3. **= 13 577 (nearest whole number)**

WORKED EXAMPLE

EXT **A car costing £15 000 loses 18% of its value each year. Find its value after 3 years.**

1. multiplier = 1 − 0.18 = **0.82**
2. $15\,000 \times 0.82^3$
3. **= £8270.52**

WORKED EXAMPLE

EXT **A culture of bacteria doubles every 3 hours. There are 500 at the start. How many are there after 12 hours?**

1. 12 ÷ 3 = **4 doublings**
2. multiplier for each doubling = 2
3. **$500 \times 2^4 = 500 \times 16 = 8000$**

6 Time and money

Time

24-hour clock: afternoon times are the 12-hour time + 12



Time taken from 08:40 to 11:15: count on



20 min + 2 h + 15 min = 2 h 35 min

In the 24-hour clock, afternoon times are the 12-hour time plus 12. To find a time taken, count on to the next hour.

- **Hours and minutes:** there are **60** minutes in an hour, not 100. 3.4 hours = 3 h + 0.4 × 60 min = **3 h 24 min**.
- **12-hour to 24-hour:** pm times add 12 (9:05 pm = 21:05). **24-hour to 12-hour:** subtract 12 after 12:59 (17:45 = 5:45 pm).
- **Time taken:** count on to a whole hour, then whole hours, then the rest.

WORKED EXAMPLE

A film starts at 19:50 and lasts 2 hours 35 minutes. When does it finish?

1. 19:50 + 10 min = 20:00
2. 20:00 + 2 h = 22:00
3. 22:00 + 25 min = **22:25**
4. **(10 + 25 = 35 minutes ✓)**

WORKED EXAMPLE

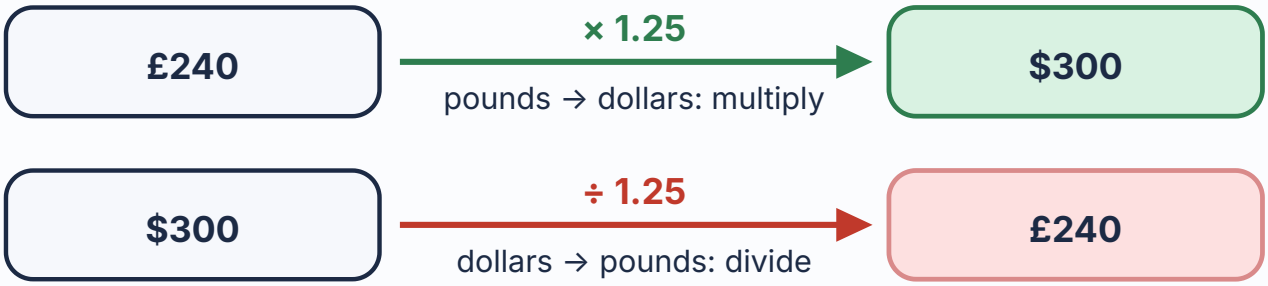
Change 0.35 hours into minutes.

1. minutes = 0.35 × 60
2. **= 21 minutes**

Riyaz Sir

Currency conversion

Exchange rate: £1 = $1.25



Sense check: here £1 buys MORE than 1 dollar, so £ → $ gives a bigger number

Multiply to change into the currency with the bigger number, divide to come back. Always do a sense check.

- Write the **rate** first (£1 = $1.25). Decide **multiply or divide**, then check that the answer is sensible.

WORKED EXAMPLE

A holiday costs €630. The rate is £1 = €1.40. How much is this in pounds?

1. euros → pounds: **divide** by 1.40
2. 630 ÷ 1.40
3. = **£450**
4. **(check: 450 × 1.40 = 630 ✓)**

WORKED EXAMPLE

A £600 laptop is bought on credit: 20% deposit and 12 payments of £42. How much extra is paid?

1. deposit = 20% of 600 = **£120**
2. payments = 12 × 42 = **£504**
3. total paid = 120 + 504 = **£624**
4. **extra = 624 − 600 = £24**

Riyaz Sir

7 Test yourself

Test yourself — cover the answers first

QUICK CHECK

1. Divide 90 in the ratio 2 : 3 : 1.
2. Simplify 45 : 60.
3. Write 2 : 5 in the form 1 : n.
4. Increase 60 by 15%.
5. Decrease 80 by 25%.
6. What percentage is 12 of 48?
7. A price rises from £80 to £92. Find the percentage increase.
8. A price after a 10% rise is £66. Find the original price.
9. Find the simple interest on £500 at 4% a year for 2 years.
10. A car travels 45 km in 30 minutes. Find its speed in km/h.
11. Change 36 km/h into m/s.
12. Write 2.4 hours in hours and minutes.
13. Write 17:45 in 12-hour time.
14. Change £80 to dollars when £1 = $1.30.
15. EXT y is directly proportional to x. y = 10 when x = 4. Find y when x = 10.
16. EXT y is inversely proportional to x. y = 2 when x = 9. Find y when x = 6.
17. EXT y is proportional to x^2. y = 18 when x = 3. Find y when x = 5.
18. EXT What multiplier means a decrease of 4.5%?
19. EXT £2000 grows by 5% a year. Find the value after 3 years.
20. EXT A population of 5000 falls by 10% each year. Find it after 2 years.

EXAM-STYLE

21. Ali, Beth and Cara share £360 in the ratio 2 : 3 : 7. Work out Cara's share. [3]
22. A coat normally costs £85. In a sale it is reduced by 24%. Find the sale price. [2]
23. The price of a TV including 20% VAT is £510. Find the price before VAT. [3]
24. A car travels 90 km in 1 hour 15 minutes. Find its average speed in km/h. [2]
25. A train leaves at 09:48 and the journey takes 3 hours 27 minutes. At what time does it arrive? [2]
26. Sam invests $4000 at 2.5% compound interest each year for 3 years. Calculate the amount at the end, to the nearest cent. [3]
27. EXT y is inversely proportional to x. y = 8 when x = 3. Find y when x = 12. [3]
28. EXT A machine costs £25 000 and loses 12% of its value each year. Find its value after 4 years, to the nearest pound. [3]
29. EXT A price falls from £2.50 to £2.20. Find the percentage decrease. [2]

Show answers

1. 30, 45, 15
2. 3 : 4
3. 1 : 2.5
4. 69
5. 60
6. 25%
7. 15%
8. £60
9. £40
10. 90 km/h
11. 10 m/s
12. 2 h 24 min
13. 5:45 pm
14. $104
15. 25
16. 3
17. 50
18. 0.955
19. £2315.25
20. 4050
21. total parts = 2 + 3 + 7 = 12
    one part = 360 ÷ 12 = 30
    Cara = 7 × 30 = £210
22. multiplier = 1 − 0.24 = 0.76
    85 × 0.76 = £64.60
23. price with VAT = 120%
    510 ÷ 1.2
    = £425
24. 1 h 15 min = 1.25 h
    90 ÷ 1.25 = 72 km/h
25. 09:48 + 12 min = 10:00
    10:00 + 3 h = 13:00, then + 15 min
    13:15
26. multiplier = 1.025
    4000×1.025^3
    = $4307.56
27. y = k ÷ x, so 8 = k ÷ 3, k = 24
    y = 24 ÷ x
    x = 12 gives y = 2

28. multiplier = 0.88

$25\,000 \times 0.88^4$

= £14 992

29. change = 0.30

$\frac{0.30}{2.50} \times 100$

= 12%

Stuck on a question?

Send it to me on WhatsApp. I read every message.

Want regular help? Send this to your parent →

Share with a friend

Ask Riyaz Sir