IGCSE MATHEMATICS 0580 · CHAPTER 10 NOTES · ~29 MIN READ

Transformations & vectors

Reflection, rotation, translation and enlargement, then vectors: column vectors, addition, magnitude and vector geometry. Every transformation is drawn on a grid so you can see exactly what moves where.

Reflection | Rotation | Translation | Enlargement | Vectors | Magnitude | Vector geometry

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No tag = **Core** (everyone) EXTENDED Extended only highlight = comes up again and again

Test yourself at the end

IN 30 SECONDS

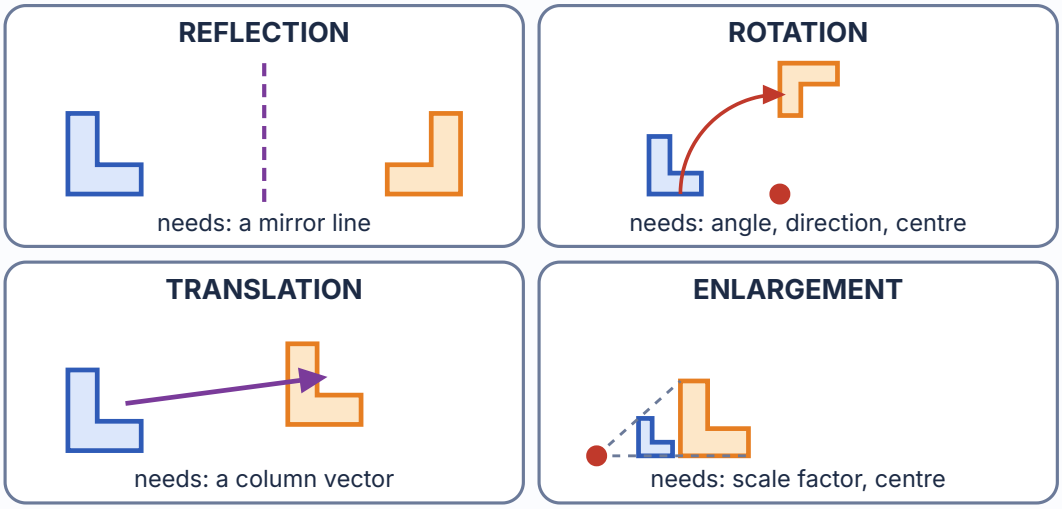
- **Four transformations: reflection** (mirror line), **rotation** (angle, direction, centre), **translation** (column vector) and **enlargement** (scale factor, centre). To **describe fully**, give **all** the details.
- **Reflection, rotation and translation** keep the shape the same size (the shapes are **congruent**). **Enlargement** changes the size, and lengths are multiplied by the **scale factor** and areas by its **square**.
- **Vectors** EXT have **size and direction**. **Add** them tip to tail, **multiply** them by numbers, find the **magnitude** with Pythagoras, and use **routes** to solve geometry problems.

BY THE END YOU CAN

- Draw and describe reflections, rotations and translations.
- Draw and describe enlargements with a positive scale factor, and use the centre of enlargement.
- Use fractional and negative scale factors, and find the area scale factor. EXT
- Write translations as column vectors and find the vector between two points.
- Add and subtract vectors, multiply by a scalar and find the magnitude of a vector. EXT
- Use vectors to solve geometry problems and to show that points are in a straight line. EXT

1 The four transformations

Four moves, four descriptions



The first three keep the size (congruent). Enlargement changes it.

Reflection, rotation and translation keep the size of the shape. Enlargement changes it.

Object and image — the **object** is the shape before the transformation. The **image** is the shape after it. Label the image points **A′, B′, C′** (A dash, B dash, C dash).

Congruent — exactly the **same shape and size**. A shape and its image are congruent after a reflection, rotation or translation, even if it is flipped or turned.

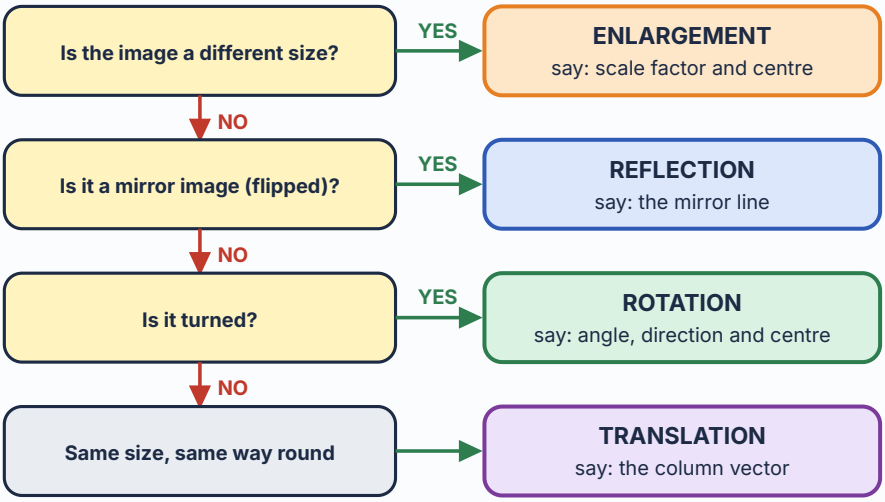
Similar — the **same shape** but a **different size**. An enlargement gives a **similar** shape.

Transformation	You must say	Shape and size
Reflection	the mirror line, such as x = 2 or y = x	congruent
Rotation	the angle, the direction and the centre	congruent
Translation	the column vector	congruent
Enlargement	the scale factor and the centre	similar

- **"Describe fully the single transformation"** means you must name the transformation **and** give **every** detail in the last column. A missing detail loses marks.
- **Never combine words.** Do not write "reflection and translation" for a single transformation. Pick one.

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Which transformation is it?



Ask the questions in this order: size, then flipped, then turned. If none of them is true, it is a translation.

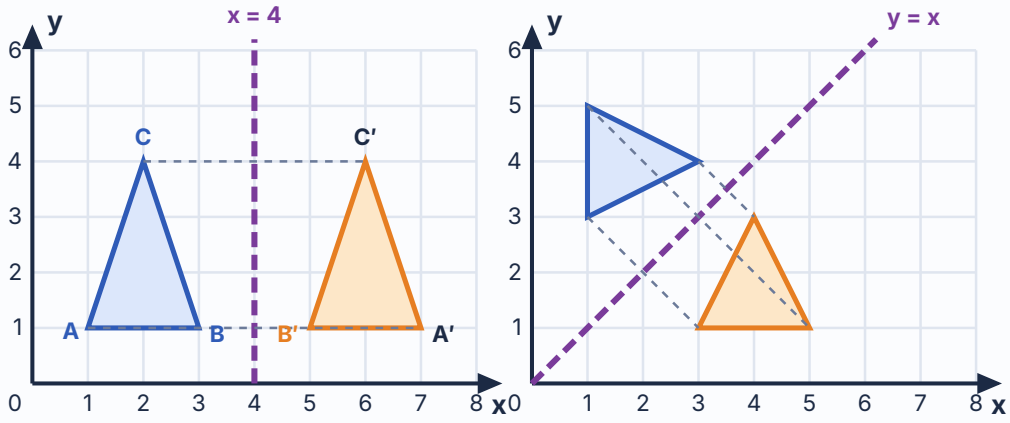
- **Compare the object and the image.** If the lengths have changed, it is an enlargement.
- **Same size but flipped?** The shape is a **mirror image**. Look for the mirror line between them.
- **Same size and the same way round?** It is either a **rotation** (the shape has turned) or a **translation** (it has just slid across). In a translation every side stays parallel to where it was.

EXAM TIP

Check with one vertex. Join a corner of the object to the same corner on the image. If the shape is a reflection, the mirror line cuts this line **in half at 90°**.

2 Reflection

Reflecting a shape



Each point and its image are the same distance from the mirror line.
The dashed lines cross the mirror line at 90°.

Measure the distance from each point to the mirror line, then go the same distance on the other side.

Mirror line — the line that the shape is reflected in. Each point and its image are the **same distance** from the mirror line, and the join between them is **perpendicular** to it.

- **Step 1:** draw a **perpendicular** from each vertex to the mirror line.
- **Step 2:** measure the distance and go the **same distance** beyond the line.
- **Step 3:** plot the new vertices and join them in the same order.
- **Points on the mirror line do not move.**
- **Finding the mirror line:** join a point to its image. The mirror line is the **perpendicular bisector** of this join, so it passes through the **midpoint** at 90°.

WORKED EXAMPLE

Reflect the triangle with vertices A(1, 1), B(3, 1) and C(2, 4) in the line x = 4.

1. A is 3 units left of the line, so A′ is 3 units right: (7, 1)
2. B is 1 unit left, so B′ = (5, 1)
3. C is 2 units left, so C′ = (6, 4)
4. **A′(7, 1), B′(5, 1), C′(6, 4)**

WORKED EXAMPLE

Triangle PQR has P(1, 3), Q(1, 5) and R(3, 4). Its image is P′(3, 1), Q′(5, 1) and R′(4, 3). Describe the transformation.

1. the x and y values have swapped in every point
2. **reflection in the line y = x**

WORKED EXAMPLE

A point P(1, 4) is mapped to P′(7, 4) by a reflection. Find the mirror line.

1. the mirror line passes through the midpoint of PP′
2. midpoint = ((1 + 7) ÷ 2, 4) = (4, 4)
3. **PP′ is horizontal, so the mirror line is vertical: x = 4**

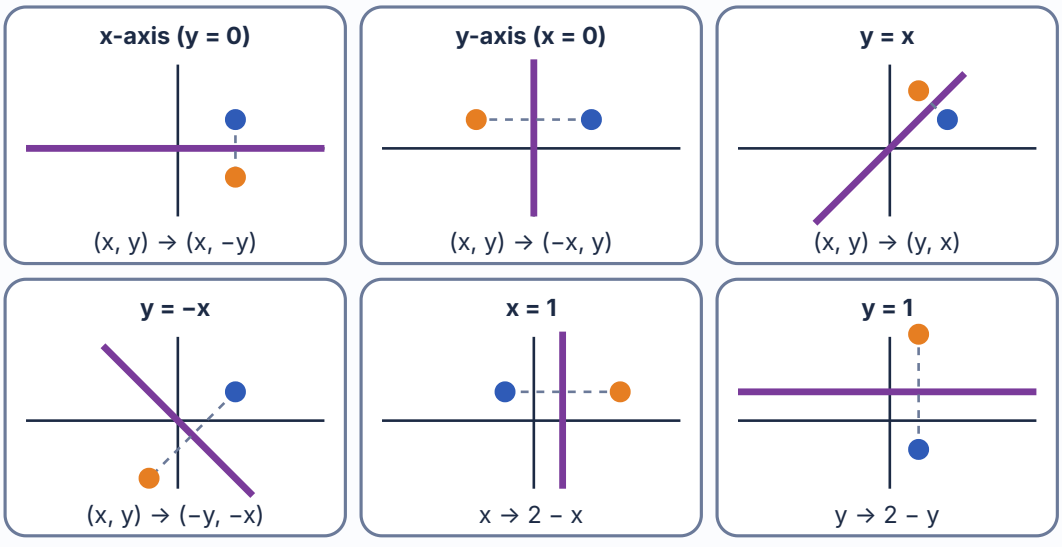
COMMON MISTAKE

Measuring the distance from the **axis** instead of from the **mirror line**. Always measure to the line you are reflecting in.

COMMON MISTAKE

Writing the mirror line as "the line through 4". Say **x = 4**: the equation, with x or y.

Mirror lines to know



For the mirror lines on the axes and the lines y = x and y = −x there is a pattern for the coordinates.

Mirror line	The point (x, y) becomes
x-axis (y = 0)	(x, −y)
y-axis (x = 0)	(−x, y)
y = x	(y, x)
y = −x	(−y, −x)

- **For x = k:** the y value stays the same and the new x value is **2k − x**.
- **For y = k:** the x value stays the same and the new y value is **2k − y**.
- **Sketch it.** If you forget the rule, draw the point and the mirror line, then count the squares.

WORKED EXAMPLE

Reflect (3, −2) in the line y = x.

1. swap the coordinates
2. **(−2, 3)**

WORKED EXAMPLE

Reflect (4, 1) in the line y = −x.

1. (x, y) becomes (−y, −x)
2. (−1, −4)
3. **(−1, −4)**

WORKED EXAMPLE

Reflect (2, 5) in the line x = −1.

1. new x = 2 × (−1) − 2 = −4
2. y stays the same
3. **(−4, 5)**

WORKED EXAMPLE

Reflect (−3, 4) in the line y = 2.

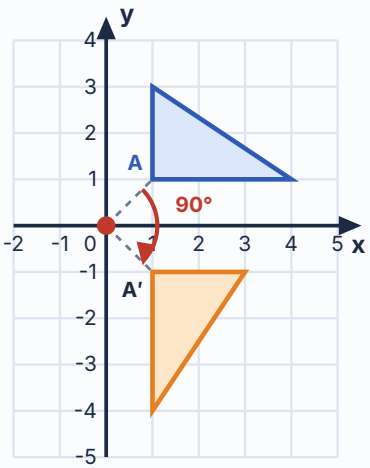
1. new y = 2 × 2 − 4 = 0
2. x stays the same
3. **(−3, 0)**

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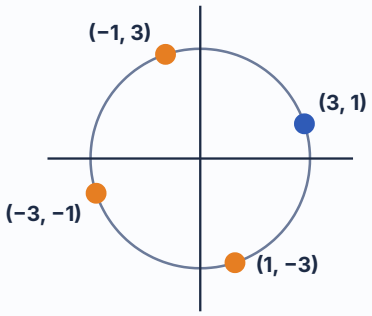
3 Rotation

Turning about a centre

90° clockwise about (0, 0)



One point, three turns



90° clockwise: (x, y) → (y, −x)
90° anticlockwise: (x, y) → (−y, x)
180°: (x, y) → (−x, −y)

A rotation turns the shape about a fixed point, the centre. The shape does not change size or shape.

Centre of rotation — the point that **does not move**. Every other point turns around it.

- **You must give three things:** the **angle** (for example 90°), the **direction** (clockwise or anticlockwise) and the **centre**.
- **Use tracing paper.** Trace the shape, put your pencil on the centre, turn the paper through the angle and draw the image.
- **For 180°** the direction does not matter. A half-turn is the same either way.
- **Rules about the origin (0, 0):** 90° clockwise takes (x, y) to **(y, −x)**. 90° anticlockwise takes (x, y) to **(−y, x)**. 180° takes (x, y) to **(−x, −y)**.
- **To find the centre:** use tracing paper, or draw the **perpendicular bisector** of two joins (A to A′ and B to B′). They meet at the centre.

WORKED EXAMPLE

Rotate the point (3, 1) through 90° clockwise about the origin.

1. (x, y) becomes (y, −x)
2. **(1, −3)**

WORKED EXAMPLE

Rotate the triangle with vertices (1, 1), (4, 1) and (1, 3) through 90° clockwise about the origin.

1. (1, 1) becomes (1, −1)
2. (4, 1) becomes (1, −4)
3. (1, 3) becomes (3, −1)
4. **(1, −1), (1, −4), (3, −1)**

WORKED EXAMPLE

Rotate (5, 2) through 180° about the point (3, 1).

1. the vector from the centre to the point is (2, 1)
2. a half-turn reverses it to (−2, −1)
3. **new point = (3 − 2, 1 − 1) = (1, 0)**

WORKED EXAMPLE

Rotate (4, 1) through 90° clockwise about (1, 1).

1. vector from the centre = (3, 0)
2. 90° clockwise turns (x, y) to (y, −x), so (3, 0) becomes (0, −3)
3. **new point = (1 + 0, 1 − 3) = (1, −2)**

WORKED EXAMPLE

The triangle (1, 1), (4, 1), (1, 3) is mapped to (1, −1), (1, −4), (3, −1). Describe the transformation.

1. the shape has turned and kept its size, so it is a rotation
2. (1, 1) to (1, −1) matches (x, y) to (y, −x)
3. **rotation, 90° clockwise, centre (0, 0)**

COMMON MISTAKE

Turning the **wrong way**. Check with one corner: which way did the corner travel?

COMMON MISTAKE

Forgetting the **centre**. "Rotation 90° clockwise" scores only part of the marks.

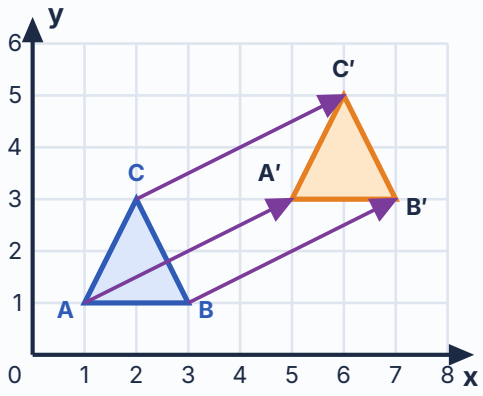
COMMON MISTAKE

Rotating about the origin when the centre is **somewhere else**. Use the vector from the centre.

4 Translation

Sliding a shape

ABC moves 4 right and 2 up



Column vector

top number: right (+) or left (−)
bottom number: up (+) or down (−)

$\begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **the move shown: 4 right, 2 up**

$\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **3 right, 2 down**

$\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ **1 left, 4 up**

A translation slides every point by the same amount. The column vector tells you how far right or left, then up or down.

Column vector — two numbers in brackets, one above the other. The **top** number is the movement in the **x direction** (right is positive) and the **bottom** number is the movement in the **y direction** (up is positive).

- **Write it as a column** $\begin{pmatrix}3\\2\end{pmatrix}$: the 3 is above the 2 inside one pair of brackets. Do **not** write it as coordinates (3, 2).
- **Every point moves the same way.** The shape is not turned and not flipped. All sides stay parallel to where they were.
- **To find the vector:** subtract the coordinates of a point from the coordinates of its image. For each point: new minus old.
- **The inverse:** to undo a translation, use the **opposite** vector. The inverse of $\begin{pmatrix}4\\2\end{pmatrix}$ is $\begin{pmatrix}-4\\-2\end{pmatrix}$.

WORKED EXAMPLE

Translate the point (2, 1) by $\begin{pmatrix}-3\\5\end{pmatrix}$.

1. x: 2 + (−3) = −1
2. y: 1 + 5 = 6
3. **(−1, 6)**

WORKED EXAMPLE

Triangle ABC has A(1, 1), B(3, 1) and C(2, 3). Find its image after translation by $\begin{pmatrix}4\\2\end{pmatrix}$.

1. A′ = (1 + 4, 1 + 2) = (5, 3)
2. B′ = (3 + 4, 1 + 2) = (7, 3)
3. C′ = (2 + 4, 3 + 2) = (6, 5)
4. **A′(5, 3), B′(7, 3), C′(6, 5)**

WORKED EXAMPLE

The point A(2, 3) is mapped to A′(7, 1) by a translation. Write down the column vector.

1. right: 7 − 2 = 5
2. up: 1 − 3 = −2
3. $\begin{pmatrix}5\\-2\end{pmatrix}$

WORKED EXAMPLE

A point P(3, 2) is reflected in the line y = x and then translated by $\begin{pmatrix}-1\\4\end{pmatrix}$. Find the final position.

1. reflect: (2, 3)
2. translate: (2 − 1, 3 + 4)
3. **(1, 7)**

COMMON MISTAKE

Mixing up the **x and y** numbers, or getting a **sign** wrong. Right and up are positive. Left and down are negative.

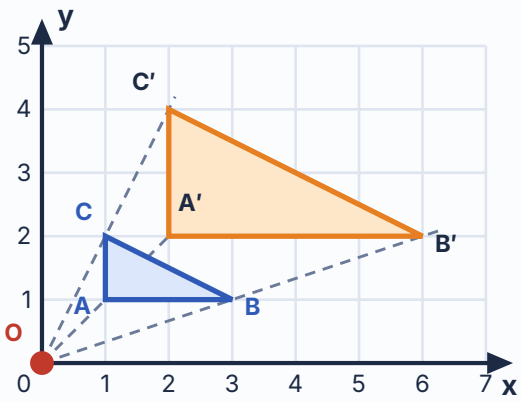
EXAM TIP

Combined transformations: do them **one at a time, in the order given**. Changing the order can change the answer.

5 Enlargement

Enlarging from a centre

Scale factor 2, centre O (0, 0)



How to enlarge

1. Draw a line from the centre through each vertex.
2. Measure the distance from the centre.
3. Multiply the distance by the scale factor.
4. Plot the new points and join.

scale factor = image ÷ object
lengths × 2
area × 2^2 = × 4

The image is similar to the object: same shape, different size.

Each image point lies on the line from the centre through the object point. The distance from the centre is multiplied by the scale factor.

Scale factor — the number that **multiplies every length**.
Scale factor = image length ÷ object length.

Centre of enlargement — the fixed point that every distance is measured from. It does not move.

- **You must give two things:** the **scale factor** and the **centre**.
- **To enlarge:** measure the distance from the centre to each vertex, multiply by the scale factor and mark the new points on the same lines.
- **The object and image are similar:** same shape, but all lengths are multiplied by the same scale factor.
- **Finding the centre:** draw a line through each object vertex and its image vertex. The lines meet at the **centre**.
- **Angles do not change.** Only lengths and areas change.

WORKED EXAMPLE

Enlarge the triangle A(1, 1), B(3, 1), C(1, 2) by scale factor 2 with centre the origin.

1. multiply each coordinate by 2
2. A′ = (2, 2), B′ = (6, 2), C′ = (2, 4)
3. **A′(2, 2), B′(6, 2), C′(2, 4)**

WORKED EXAMPLE

Enlarge the point (3, 2) by scale factor 3 with centre (1, 1).

1. vector from the centre = (3 − 1, 2 − 1) = (2, 1)
2. times 3 = (6, 3)
3. **image = (1 + 6, 1 + 3) = (7, 4)**

WORKED EXAMPLE

Triangle A(2, 1), B(3, 1), C(2, 2) maps to A′(4, 3), B′(7, 3), C′(4, 6). Describe the transformation.

1. AB = 1 and A′B′ = 3, so the scale factor is 3
2. line through A and A′: $y = x - 1$
3. line through B and B′: $y = 0.5x - 0.5$
4. they meet when $x - 1 = 0.5x - 0.5$, so $x = 1$ and $y = 0$
5. **enlargement, scale factor 3, centre (1, 0)**

WORKED EXAMPLE

A triangle of area 5 cm² is enlarged by scale factor 3. Find the area of the image.

1. area scale factor = $3^2 = 9$
2. **$5 \times 9 = 45$ cm²**

COMMON MISTAKE

Giving the **scale factor as a difference** (adding 2 to each length) instead of a **multiplier**.

COMMON MISTAKE

Measuring distances from a **corner of the shape** instead of from the **centre**.

COMMON MISTAKE

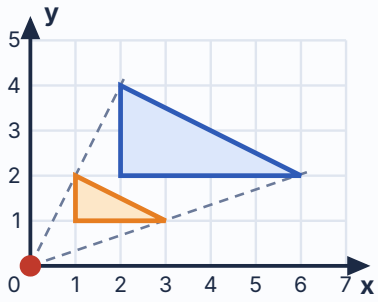
Thinking the **area** is multiplied by the scale factor. Areas are multiplied by the **scale factor squared**, and volumes by the scale factor **cubed**.

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Fractional and negative scale factors

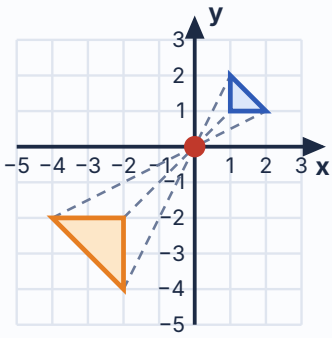
EXTENDED

Scale factor ½
image is smaller



Same side of the centre, half the distance away.

Scale factor −2
image is on the other side



Opposite side of the centre, twice the distance, upside down.

Always measure from the centre of enlargement.

A fraction makes the image smaller. A negative number puts the image on the opposite side of the centre.

- **Scale factor between 0 and 1** (such as ½): the image is **smaller** and **closer** to the centre, on the same side.
- **Negative scale factor** (such as −2): the image is on the **opposite side** of the centre, the **same ratio of distance** and **upside down** (rotated 180°).
- **The size depends on the number without its sign.** A scale factor of −2 makes the image twice as big, just on the other side.
- **To describe a reduction**, give the fraction. If the image is half the size, the scale factor is ½.

WORKED EXAMPLE

Enlarge the triangle (2, 2), (6, 2), (2, 4) by scale factor ½ with centre the origin.

1. halve every coordinate
2. **(1, 1), (3, 1), (1, 2)**

WORKED EXAMPLE

Enlarge the triangle (1, 1), (2, 1), (1, 3) by scale factor −2 with centre the origin.

1. multiply every coordinate by −2
2. (1, 1) to (−2, −2)
3. (2, 1) to (−4, −2)
4. (1, 3) to (−2, −6)
5. **(−2, −2), (−4, −2), (−2, −6)**

WORKED EXAMPLE

Enlarge the point (3, 2) by scale factor −2 with centre (1, 1).

1. vector from the centre = (2, 1)
2. times −2 = (−4, −2)
3. **image = (1 − 4, 1 − 2) = (−3, −1)**

WORKED EXAMPLE

A shape has area 12 cm². It is enlarged by scale factor 2.5. Find the area of the image.

1. area scale factor = 2.5^2 = 6.25
2. **12 × 6.25 = 75 cm²**

EXAM TIP

Check the side. For a negative scale factor, the object and image are on **opposite sides** of the centre. For a positive one they are on the **same** side.

Column vectors and multiples

EXTENDED

a



$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

3 right, 2 up

−a

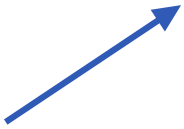


$\begin{pmatrix} -3 \\ -2 \end{pmatrix}$

3 left, 2 down

same length, opposite direction

2a



$\begin{pmatrix} 6 \\ 4 \end{pmatrix}$

6 right, 4 up

same direction, twice as long

Parallel vectors

one is a multiple of the other
k a: multiply both numbers by k
k > 0 same direction
k < 0 opposite direction

A vector has size and direction. Multiplying by a number stretches it, and a negative number reverses it.

Vector — a quantity with both **size and direction**, shown as an arrow. Written as a **bold letter** (a), as a letter with a line under it, or as **AB** for the arrow from A to B.

Scalar — an ordinary number. Multiplying a vector by a scalar **k** multiplies **both** numbers in its column by k.

- **Components:** the column vector $\begin{pmatrix}3\\2\end{pmatrix}$ means 3 to the right and 2 up.
- **Multiplying:** 2a means a twice as long in the same direction. −a means the same length in the **opposite** direction.
- **Parallel vectors** are **multiples** of each other. $\begin{pmatrix}6\\4\end{pmatrix}$ is parallel to $\begin{pmatrix}3\\2\end{pmatrix}$ because it is 2 times as big.
- **Vector from A to B:** subtract the coordinates, **AB = end minus start**. BA is the **opposite**: BA = −AB.
- **Position vector:** the vector from the origin O to a point. The position vector of A(2, 3) is OA = $\begin{pmatrix}2\\3\end{pmatrix}$.

Operation	Example	Result
Add	$\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}1\\4\end{pmatrix}$	$\begin{pmatrix}4\\6\end{pmatrix}$
Subtract	$\begin{pmatrix}3\\2\end{pmatrix} - \begin{pmatrix}1\\4\end{pmatrix}$	$\begin{pmatrix}2\\-2\end{pmatrix}$
Scalar multiple	$2 \times \begin{pmatrix}3\\2\end{pmatrix}$	$\begin{pmatrix}6\\4\end{pmatrix}$

WORKED EXAMPLE

Find the column vector AB when A is (2, 1) and B is (7, 4).

1. AB = B − A = $\begin{pmatrix}7-2\\4-1\end{pmatrix}$
2. $\begin{pmatrix}5\\3\end{pmatrix}$ **(BA would be** $\begin{pmatrix}-5\\-3\end{pmatrix}$**)**

WORKED EXAMPLE

Given a = $\begin{pmatrix}3\\2\end{pmatrix}$ **and b =** $\begin{pmatrix}1\\4\end{pmatrix}$**, find 3a − b.**

1. 3a = $\begin{pmatrix}9\\6\end{pmatrix}$
2. 3a − b = $\begin{pmatrix}9-1\\6-4\end{pmatrix}$
3. $\begin{pmatrix}8\\2\end{pmatrix}$

WORKED EXAMPLE

Find the value of k if $\begin{pmatrix}k\\6\end{pmatrix}$ **is parallel to** $\begin{pmatrix}2\\3\end{pmatrix}$**.**

1. the bottom number has been multiplied by 6 ÷ 3 = 2
2. so the top number is multiplied by 2 as well
3. **k = 2 × 2 = 4**

COMMON MISTAKE

Writing AB as **start minus end**. It is **end minus start**.

COMMON MISTAKE

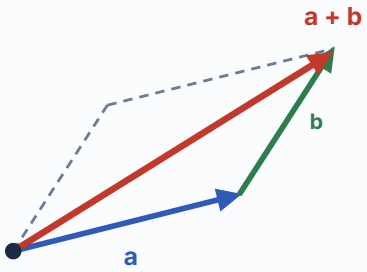
Writing a vector as coordinates (3, 2). **Coordinates** give a position. A **column vector** gives a movement.

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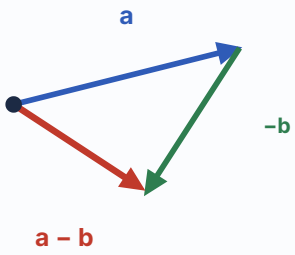
Adding and subtracting vectors

EXTENDED

Add: tip to tail



Subtract: add the opposite



$$\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix} \qquad \begin{pmatrix} 3 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$$

Add or subtract the top numbers, then the bottom numbers.

To add, put the vectors tip to tail. The sum goes from the start to the end. To subtract, add the opposite.

- **Add component by component:** the top numbers together and the bottom numbers together.
- **Tip to tail:** the second vector starts where the first one ends. The **resultant** goes straight from the very start to the very end.
- **Subtracting b** is the same as **adding −b**, the vector with the same length in the opposite direction.
- **AB + BC = AC.** The two middle letters cancel.

WORKED EXAMPLE

Given $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find $\mathbf{p} + \mathbf{q}$ and $2\mathbf{p} - \mathbf{q}$.

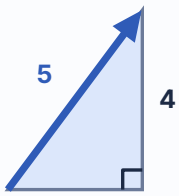
1. $\mathbf{p} + \mathbf{q} = \begin{pmatrix} 2 - 3 \\ -1 + 4 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
2. $2\mathbf{p} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$
3. $2\mathbf{p} - \mathbf{q} = \begin{pmatrix} 4 + 3 \\ -2 - 4 \end{pmatrix} = \begin{pmatrix} 7 \\ -6 \end{pmatrix}$

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Magnitude of a vector

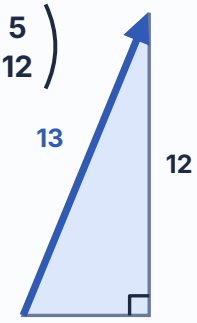
EXTENDED

$\begin{pmatrix} 3 \\ 4 \end{pmatrix}$



$|a| = \sqrt{(3^2 + 4^2)}$
$= \sqrt{25} = 5$

$\begin{pmatrix} 5 \\ 12 \end{pmatrix}$



$|b| = \sqrt{(5^2 + 12^2)}$
$= \sqrt{169} = 13$

Magnitude (length)

for a column vector with x on top and y below:
$|a| = \sqrt{(x^2 + y^2)}$
The signs do not matter, because squaring removes minus signs.

top −6, bottom 8:
$\sqrt{(36 + 64)} = \sqrt{100} = 10$

Pythagoras gives the length of any vector.

The components are the two shorter sides of a right-angled triangle. The magnitude is the hypotenuse.

Magnitude — the **length** of a vector. For $a = \begin{pmatrix} x \\ y \end{pmatrix}$ it is **$|a| = \sqrt{(x^2 + y^2)}$**. For the arrow from A to B it is written **$|AB|$**.

- **Always square first.** The minus signs vanish when you square, so you never get a negative magnitude.
- **A common pattern:** $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ has magnitude 5, and $\begin{pmatrix} 5 \\ 12 \end{pmatrix}$ has magnitude 13.
- **Distance between two points:** $|AB| = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$.

WORKED EXAMPLE

Find the magnitude of $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$, to 3 significant figures.

1. $|a| = \sqrt{(2^2 + (-3)^2)}$
2. $= \sqrt{(4 + 9)} = \sqrt{13}$
3. **$= 3.61$**

WORKED EXAMPLE

Find the column vector from A(1, 2) to B(4, 6) and its length.

1. $AB = \begin{pmatrix} 4 - 1 \\ 6 - 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$
2. **$|AB| = \sqrt{(9 + 16)} = \sqrt{25} = 5$**

WORKED EXAMPLE

Given $p = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $q = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, find $|p + q|$.

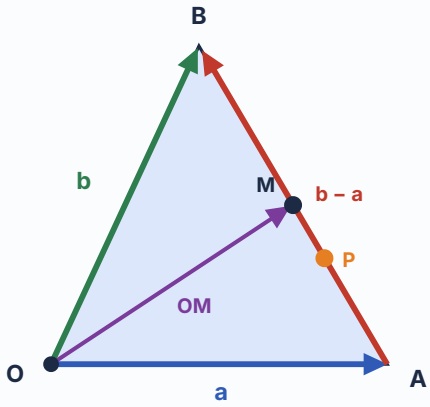
1. $p + q = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$
2. **$|p + q| = \sqrt{((-1)^2 + 3^2)} = \sqrt{10} = 3.16$**

COMMON MISTAKE

Adding the lengths. **$|a + b|$** is **not** $|a| + |b|$. Add the **vectors** first and then find the length.

Vector geometry

EXTENDED



OA = a and OB = b

AB = AO + OB = −a + b = b − a
M is the midpoint of AB, so AM = ½AB
OM = OA + AM = a + ½(b − a)
= ½a + ½b
P divides AB so that AP : PB = 1 : 2
AP = ⅓AB, so OP = a + ⅓(b − a)
= ⅔a + ⅓b
Method: go from the start to the end by a route you know. Going against an arrow means minus.

Use a route you know. Going along an arrow is plus, and going against an arrow is minus.

- **Step 1:** write down the vectors you are given in terms of **a** and **b**.
- **Step 2:** find a **route** from the start to the end that uses only these vectors. Reverse any arrow you go against.
- **Step 3: simplify** by collecting the a terms and the b terms.
- **For the midpoint** of AB: AM = ½AB. For a point P with **AP : PB = 1 : 2**, AP = ⅓AB.
- **With O as the origin:** AB = OB − OA = b − a. This is the pattern that appears most often.

WORKED EXAMPLE

OA = a and OB = b. M is the midpoint of AB. Find AB and OM.

1. AB = AO + OB = −a + b = **b − a**
2. AM = ½(b − a)
3. **OM = OA + AM = a + ½(b − a) = ½a + ½b**

WORKED EXAMPLE

OABC is a parallelogram with OA = a and OC = c. Find OB and the vector from A to M, where M is the midpoint of AB.

1. OB = OA + AB, and AB = OC = c, so OB = **a + c**
2. **AM = ½AB = ½c**

WORKED EXAMPLE

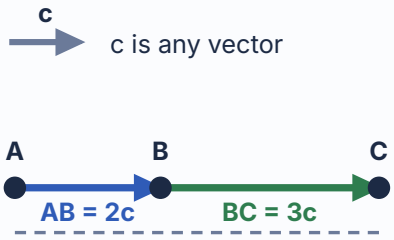
In the same parallelogram, P lies on OB with OP : PB = 2 : 3. Find OP.

1. OP is 2 parts out of 5
2. **OP = ⅖OB = ⅖(a + c)**

Parallel vectors and collinear points

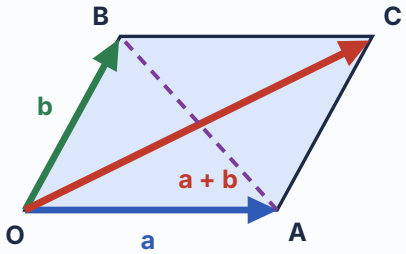
EXTENDED

Collinear points



AB and BC are parallel (both multiples of c) and they share the point B. So A, B and C lie on one straight line.

Parallelogram



OC = a + b (diagonal), AB = b − a

If two vectors are multiples of each other they are parallel. If they also share a point, the points are on one straight line.

Collinear — lying on the **same straight line**.

- **To show A, B and C are collinear:** show that AB is a **multiple** of BC (they are parallel) and that they share the point **B**.
- **In a parallelogram** with OA = a and OB = b: the diagonal OC = a + b, and the other diagonal AB = b − a.
- **Opposite sides of a parallelogram** are equal vectors: AC = OB and BC = OA.

WORKED EXAMPLE

Given AB = 4p − 2q and BC = 6p − 3q, show that A, B and C are collinear.

1. BC = 6p − 3q = 1.5 × (4p − 2q) = 1.5AB
2. so BC is parallel to AB
3. **they share the point B, so A, B and C are on one straight line**

WORKED EXAMPLE

OABC is a parallelogram with OA = a and OC = c. M is the midpoint of AB. P is the point on OB with OP = ⅔OB. Show that P lies on CM.

1. OB = a + c, so OP = ⅔(a + c)
2. OM = a + ½c, so CM = −c + a + ½c = a − ½c
3. CP = −c + ⅔(a + c) = ⅔a − ⅓c = ⅔(a − ½c) = ⅔CM
4. **CP is a multiple of CM and they share C, so P lies on CM**

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7 Test yourself

Test yourself — cover the answers first

QUICK CHECK

1. Reflect the point (3, 5) in the x-axis.
2. Reflect the point (3, 5) in the y-axis.
3. Reflect the point (2, 7) in the line y = x.
4. Reflect the point (4, −1) in the line x = 2.
5. Reflect the point (1, 6) in the line y = 3.
6. Rotate the point (2, 3) through 90° clockwise about the origin.
7. Rotate the point (4, −1) through 180° about the origin.
8. Rotate the point (5, 2) through 90° anticlockwise about the origin.
9. Translate the point (3, 4) by the vector $\begin{pmatrix} 2 \\ -5 \end{pmatrix}$.
10. The point A(1, 2) is mapped to A′(6, 0) by a translation. Write down the column vector.
11. What must you state to describe a rotation fully?
12. What must you state to describe an enlargement fully?
13. A shape is enlarged by scale factor 3. A side of length 4 cm becomes how long?
14. A triangle of area 5 cm² is enlarged by scale factor 3. Find the area of the image.
15. Enlarge the point (2, 1) by scale factor 2 with centre the origin.
16. EXT Enlarge the point (6, 4) by scale factor ½ with centre the origin.
17. EXT Enlarge the point (2, 3) by scale factor −1 with centre the origin.
18. EXT Enlarge the point (3, 2) by scale factor 3 with centre (1, 1).
19. EXT Given a = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and b = $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$, find a + b.
20. EXT Given a = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and b = $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$, find 3a.
21. EXT Given a = $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and b = $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$, find a − b.
22. EXT Find the magnitude of the vector $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$.
23. EXT Find the column vector AB and its length when A is (1, 2) and B is (4, 6).
24. EXT Are the vectors $\begin{pmatrix} 2 \\ 6 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ parallel? Give a reason.
25. EXT OA = a and OB = b. Write AB in terms of a and b.
26. EXT M is the midpoint of AB, where OA = a and OB = b. Write OM in terms of a and b.

EXAM-STYLE

27. Triangle T has vertices (1, 1), (3, 1) and (3, 4). (a) Reflect T in the y-axis. (b) Translate T by $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$. Write down the vertices of each image. [4]
28. Triangle ABC has A(1, 1), B(4, 1) and C(1, 3). Triangle A′B′C′ has A′(1, −1), B′(1, −4) and C′(3, −1). Describe fully the single transformation that maps ABC onto A′B′C′. [3]
29. Triangle PQR has P(1, 3), Q(1, 5) and R(3, 4). Its image is P′(3, 1), Q′(5, 1) and R′(4, 3). Describe fully the single transformation. [2]
30. Triangle ABC has A(2, 1), B(3, 1) and C(2, 2). It is enlarged by scale factor 3 with centre (1, 0). Find the coordinates of the image and the ratio of the areas. [4]

31. A point P(3, 2) is reflected in the line y = x. The image is then translated by $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$. Find the coordinates of the final image. [2]

32. EXT Triangle (1, 1), (2, 1), (1, 3) is enlarged by scale factor −2 with centre the origin. Write down the vertices of the image. [3]

33. EXT $\mathbf{p} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$. Find $\mathbf{p} + 2\mathbf{q}$ and $|\mathbf{p} + 2\mathbf{q}|$ to 3 significant figures. [4]

34. EXT A is (−1, 2) and B is (5, 10). Find AB, |AB| and the midpoint of AB. [4]

35. EXT OABC is a parallelogram with OA = a and OC = c. M is the midpoint of AB. Find (a) OB, (b) OM. Then show that the point P on OB with OP = ⅔OB lies on the line CM. [6]

36. EXT AB = 4p − 2q and BC = 6p − 3q. Show that A, B and C lie on a straight line. [3]

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Show answers

1. (3, −5)
2. (−3, 5)
3. (7, 2)
4. (0, −1)
5. (1, 0)
6. (3, −2)
7. (−4, 1)
8. (−2, 5)
9. (5, −1)
10. $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$
11. the angle, the direction and the centre
12. the scale factor and the centre
13. 12 cm
14. 45 cm²
15. (4, 2)
16. (3, 2)
17. (−2, −3)
18. (7, 4)
19. $\begin{pmatrix} 1 \\ 7 \end{pmatrix}$
20. $\begin{pmatrix} 6 \\ 9 \end{pmatrix}$
21. $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$
22. 10
23. AB = $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and $|AB| = 5$
24. Yes, $\begin{pmatrix} 3 \\ 9 \end{pmatrix} = 1.5 \times \begin{pmatrix} 2 \\ 6 \end{pmatrix}$
25. b − a
26. ½(a + b)
27. (a) (−1, 1), (−3, 1), (−3, 4)
    (b) (1 − 2, 1 + 3) = (−1, 4)
    (3 − 2, 1 + 3) = (1, 4)
    (3 − 2, 4 + 3) = (1, 7)
28. the triangle has turned without changing size, so it is a rotation
    (x, y) goes to (y, −x)
    rotation, 90° clockwise, centre (0, 0)
29. the x and y coordinates swap in every point
    reflection in the line y = x

30. A′ = (1, 0) + 3 × (1, 1) = (4, 3)
    B′ = (1, 0) + 3 × (2, 1) = (7, 3)
    C′ = (1, 0) + 3 × (1, 2) = (4, 6)
    area ratio = $3^2 = 9 : 1$

31. reflection gives (2, 3)
    translation gives (2 − 1, 3 + 4) = (1, 7)

32. multiply each coordinate by −2
    (−2, −2)
    (−4, −2)
    (−2, −6)

33. $2\mathbf{q} = \begin{pmatrix} -2 \\ 10 \end{pmatrix}$
    $\mathbf{p} + 2\mathbf{q} = \begin{pmatrix} 3 - 2 \\ -2 + 10 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$
    $|\mathbf{p} + 2\mathbf{q}| = \sqrt{(1 + 64)} = \sqrt{65} = 8.06$

34. $\mathrm{AB} = \begin{pmatrix} 5 - (-1) \\ 10 - 2 \end{pmatrix} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$
    $|\mathrm{AB}| = \sqrt{(36 + 64)} = 10$
    midpoint = ((−1 + 5) ÷ 2, (2 + 10) ÷ 2) = (2, 6)

35. (a) OB = a + c
    (b) OM = a + ½c
    CM = −c + a + ½c = a − ½c
    CP = −c + ⅔(a + c) = ⅔a − ⅓c = ⅔(a − ½c) = ⅔CM
    CP is a multiple of CM and they share C, so P lies on CM

36. BC = 6p − 3q = 1.5(4p − 2q) = 1.5AB
    so AB and BC are parallel
    they share the point B, so A, B and C are collinear

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